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PLASMA JET PIERCING OF MAGNETIC FIELDS AND ENTROPY TRAPPING INTO A CONSERVATIVE SYSTEM*

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To obtain a thermonuclear plasma inside a magnetic confinement system, we have either (a) to create the plasma in situ from cold gas by ionization and heating or (b) build up the plasma piecemeal by injection from some accelerator or hydromagnetic gun.

Both of these methods have characteristic requirements: For (a) as in the pinch effect, great input power is needed in order to get through the strongly radiating partly ionized plasma phase and after that it is difficult to raise temperatures of the ions much above a kilovolt by Joule heating on account of the high electrical conductivity of the plasma and the slow exchange rate between the electrons which receive the Joule heating and the ions which it is desired to heat. For (b) single particles cannot of course, be trapped into a static magnetic field and some method of circumventing the conservative property has to be found. Trapping by means of a rapidly operating magnetic trap door has been discussed by Post. Another nonadiabatic process can be the breakup of an energetic molecular ion such as $D_2^+ \rightarrow D^+ + D^+ + e$ by collision with an internal arc column as in the DCX experiment (Luce) or by collision with residual gas as in the OGRA experiment (Golovin). Energetic neutral D_2 can in principle be used instead of D_2^+. Still another process can be by interparticle collisions, as for example the injection of plasmoids by Bostick. Such plasmoids soak into the magnetic field and after developing the appropriate Hall polarization can continue across it. In this note, we propose a new method of achieving the nonadiabatic process which may prove very advantageous.

Consider a collisionless plasma of pressure p, confined on all sides by a magnetic field of magnitude $(8\pi p)^{1/2}$. Let the plasma now assume the improbable but possible configuration in which the particles all are directed at some small region on the wall. We see that the particle pressure there becomes greater than the magnetic field pressure, so that the field can be forced aside letting plasma escape. If now we review these events in reverse time sequence, a method emerges for the trapping of plasma into a magnetic field.

Consider the interaction of a broad parallel jet of cold plasma, density ρ_0, velocity v_0 in the x direction, electron mass m, ion mass M, incident on a magnetic field of magnitude B of large radial extent. The velocity v_0 is large, such that the cross section for ion-electron collisions with relative velocity v_0 is negligible (no collisions). These conditions resemble those for the plasma-field sheath described in the infinite-conductivity theory of the pinch (Rosenbluth and Garwin)[1] except that the particles here may be incident at angles other than normal to the magnetic field direction. According to this model, charge separation occurs, setting up an electric field which retards the ions and speeds up the electrons. The deflection of the ions is predominantly electrostatic, so that they are brought nearly to rest; the electrons reach an energy close to the original ion energy in the jet, and provide most of the electric current which screens off the magnetic field. The thickness of the sheath over

which the magnetic field and plasma density drop reciprocally to zero is given by $d=v_0c(mM)^{1/2}/eB$. Particles enter and leave the sheath at the same angle.

We choose the stagnation pressure of the jet, $\rho_0 v_0^2$, to be greater than the magnetic field pressure so that yielding of the field occurs. (For simplicity in the order-of-magnitude discussion which follows, the field is chosen of large extent so that the magnetic field cannot pile up to larger values.) Let us consider the situation where the jet has pierced the magnetic field for some distance. We shall assume that the process of reflection off the curved end of the cavity can be reasonably approximated by uniform (cosine law) scattering from a plane end.

Let the rate of elongation of the cavity ("tip speed") be v. Writing the pressure balance equation for the end [remembering that for cosine scattering the mean backward particle velocity is one-half the particle speed $(v_0 - v)$ relative to the wall], then

$$\tfrac{3}{2}\rho_0(v_0 - v)^2 = B^2/8\pi,$$

or

$$\epsilon = \frac{\text{tip speed}}{\text{jet speed}} = 1 - \left(\frac{B^2}{12\pi\rho_0 v_0^2}\right)^{1/2} = 1 - B/B_c;$$

$$B_c^2 = 12\pi\rho_0 v_0^2 = 7.7\times 10^2 j\sqrt{E}, \qquad (1)$$

where j = deuteron equivalent current density in amp/cm^2 and E = deuteron energy in ev. Similar equations were derived for the neutral stream magnetic storm theory of Chapman and Ferraro[2] in 1930. We see that a critical value B_c of the magnetic field will bring ϵ and the penetration velocity to zero. Larger B give a negative ϵ, corresponding to extrusion of a previously penetrated beam. Using mass conservation and letting ρ be the internal density after scattering, we next write

$$\rho_0 v_0 = \rho[\tfrac{1}{2}(v_0 - v) - v] + \rho v,$$

or

$$\rho = 2\rho_0 v_0/(v_0 - v). \qquad (2)$$

The radial pressure on the walls, holding apart the magnetic field, using this density and the mean radial velocity $\frac{1}{2}(v_0 - v)$, turns out to be $\rho_0 v_0(v_0 - v)$ which is only $2v_0/3(v_0 - v)$ of the required value of $B^2/8\pi$. To meet this difficulty, (a) the cavity could expand leaving a neck at entry—this neck reflects back some of the outwardly escaping particles raising ρ to the value required to keep the walls from collapsing, or (b) the walls could contract to the point where they intercept some of the incoming beam, again raising the wall pressure. Resorting to the analogy of the cavitation of water by a jet of air, we find both cases reported in the literature of hydrodynamics together with a third possibility—Rayleigh-Taylor instability with mixing. We may expect the latter to occur in magnetic field piercing especially when the magnetic field direction is transverse to the jet direction, since in such a case the magnetic field-plasma boundary is necessarily curved in the unstable sense.[3] Such instabilities will incidentally trap plasma and may be responsible for the trapping reported by Coensgen and Ford.[4] For injection in the axial direction, the possibility of hydromagnetic stability appears to be greater; Helmholtz instability is minimized by the highly supersonic state of the jet.[5] It might be argued that one kind of trapping is as good as another, and that this process can be welcomed; as will appear however, we require the plasma to attain the maximum possible density with a minimum of intermixed magnetic field.

In order to exploit the trapping possibilities, consider a beam entering a magnetic field such as a picket fence or cusped geometry[6] which after the initial increase to some value less than B_c, falls again. The beam can pierce the barrier forming a neck but in doing so must raise its pressure to a value larger than the field pressure further on. Consequently the jet cavity can expand against the field further down stream, forming a balloon (Fig. 1). Any jet which makes a clear traverse of the peak field without deflection, can be scattered off a $B>B_c$ "anvil" field further on, though this is not essential to the argument since scattering at the entry reduces the beam piercing power. The size of the plasma balloon will be governed by the balance between input flow and loss by back flow and diffusion into the magnetic field.

If the jet is now stopped, pressures fall and as the magnetic walls close in, the neck closes in faster, pinching off the balloon and leaving a plasma bubble trapped inside. The picket fence seems particularly well adapted to this method, since the field has the appropriate valley in the center of the machine, though the method seems adaptable with trivial modifications to Stellarator and mirror machine geometries.[7] Two types of

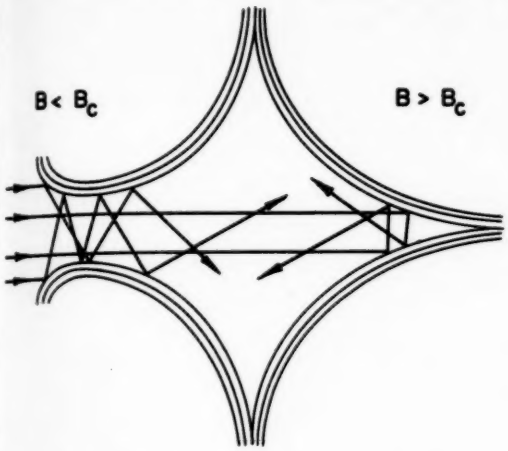

FIG. 1. Jet piercing into a picket fence magnetic field.

operation come to mind: (1) pulsed inflation whereby the plasma bubble pinched off during the off-beam period by the collapsing walls of the entry channel is left to react inside the machine; (2) steady operation whereby a large volume is kept full against losses by mixing and back-streaming. In the latter case, the prime consideration for a thermonuclear reactor becomes the chance of an ion reacting compared with its chance of finding the exit channel and escaping. At first sight it might appear that this ratio might be made arbitrarily advantageous by reducing the jet to needle-like dimensions as from an accelerator. However, there is a limit to the narrowness of the beam set by (1) breakdown of the infinite plane sheath approximation and (2) large pressure balance errors when the sheath thickness d becomes comparable with the jet diameter; and furthermore boundary layer instability may erode the jet during the penetration, limiting the ratio of the penetration depth of the unscattered beam to the jet diameter to some constant value as in the hydrodynamic analog.

The expression for the total current carried by one kind of particle in a deuterium jet of cross-sectional area $\alpha^2 d^2$, using the current density from Eq. (1), turns out to be (independently of B)

$$I = 1.35\times10^{-15}\alpha^2 mc^2\sqrt{E}, \quad (E \text{ in ev}).$$

We do not yet know how low α can be. Making a reasonable choice of $\alpha = 10$, we have

$$I = 0.3\sqrt{E} \text{ amperes}.$$

We are naturally interested in injecting plasmas for which no further heating is required, and we see that for $E = 90$ kev, $I \sim 100$ amperes. This is a current outside present accelerator practice by a factor of 100 but should not be too difficult, if one uses a condenser power supply. More immediately available seems to be a hydromagnetically accelerated plasma jet. Taking tentative estimates of particle density 10^{16} cm^{-3}, $v_0 = 10^7$ cm/sec for quantities already achieved,[8] we find the critical magnetic field for piercing to be a sizable 6500 gauss. This hydromagnetic gun already produces a volume of jet adequate for inflating several liters to a density of 10^{16} cm^{-3}. A 30-fold increase of the present velocity would bring the plasma to the optimum temperature for a DT thermonuclear reactor.

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[7]The picket fence geometry, long known to have great hydromagnetic stability, has received little experimental study mainly because of its obvious disadvantage with respect to the other main geometries, pinch, Stellarator, mirror machine, of leakier confinement. The prediction of strong cyclotron radiation from a hot plasma containing a magnetic field by B. A. Trubnikov and V. S. Kudryavtsev [Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1959), Vol. 31, p.93], may make it of primary importance to avoid this radiation by using plasmas containing zero or little magnetic field. Only the picket fence has enough hydromagnetic stability to confine such a plasma. The other devices are theoretically limited to β (= plasma pressure/magnetic field pressure) less than about 0.1.

[8]J. Marshall (to be published).

FERMI ACCELERATION OF ELECTRONS IN THE OUTER VAN ALLEN BELT*

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The persistence of the inner Van Allen belt is well understood to be due to the constancy of the strong geomagnetic field close to the earth. The weak field in the outer belt, however, is violently disturbed about once a month by a large solar storm cloud ejected by the sun in association with a flare. One can show that this solar wind should in one way or another wipe out any radiation originally present in the outer belt. In order therefore to account for the presence of this intense radiation it has been suggested that it is originally trapped in the cloud and deposited in the earth's field during the geomagnetic storm.[1] It will be shown here that this assumption is unnecessary since the radiation in the outer belt may be accounted for by a local Fermi acceleration created by the interaction between the storm and the geomagnetic field. Parker has shown that this interaction gives rise to instabilities which are most pronounced at the smaller wavelengths. Thus, in the region where the energy density of the storm and the earth's field are equal, turbulence should consist mostly of eddies of the smallest possible wavelength, i.e., a proton Larmor circumference.[2] While the storm is under way, we may expect that closed magnetic loops will frequently be broken off by Sweet's mechanism.[3] The electrons trapped in these loops will be able to undergo a Fermi acceleration without being absorbed in the earth's atmosphere as would quickly happen if they were trapped on lines connected to the earth. It is this circumstance which permits them to reach very high energies. Protons, on the other hand, are not effectively trapped in the loops because of their twisty structure: a twist should have a length of the order of a proton Larmor radius and therefore scatter protons magnetically. Thus we do not expect the proton energies to reach the Mev range through a local Fermi acceleration. Instead, protons will diffuse and be gradually exchanged with others, lobbed upward from the top of the atmosphere.[4] The loops remaining after the storm has passed will be enmeshed in the geomagnetic field and will eventually decay, depositing their electron radiation into this field where it will remain trapped by the mirror mechanism until the next storm destroys it by dumping it upon the earth through a Fermi acceleration along the geomagnetic lines of force.

In order to show that this mechanism can really work, one must (1) estimate the magnitude of the Fermi acceleration and (2) take proper account of the collisions between electrons which tend to work against it. This has been done by setting up and solving a suitable Fokker-Planck equation.

(1) For a proper treatment of Fermi acceleration one is required in general to take account of the reaction stress of the flux of accelerated particles upon the plasma adhering to the fluctuating lines of force.[5] This makes it possible to obtain an equilibrium energy spectrum for the fast particles. In the present instance we may assume that this equilibrium is frozen in when the Fermi acceleration produced by the departing cloud has become so weak that the acceleration time scale for typical Van Allen electrons is of the order of the decay time of the storm. We may take this as about a half-day (4×10^4 sec). In this time a slow electron reaches on the average a speed of 10^{10} cm/sec (~30 kev), corresponding to a Fermi acceleration, $\phi = 2.5\times10^5$ cm/sec^2.

(2) The equation of motion for an electron of momentum p in a steeply helical path around a moving line of force is

$$\dot{p} = \Omega p, \quad \Omega = \vec{\alpha}\cdot\left(\frac{\vec{B}}{B}\cdot\vec{\nabla}\frac{\vec{B}}{B}\right),$$

$\vec{\alpha}$ being the electric drift velocity and $\vec{B}$ the magnetic field. Defining $E = p^2/2m$, which is the energy at moderate velocities, we find, if the magnetic line moves in a random manner,

$$f_1 = \lim_{\Delta t\to 0}\frac{\langle\Delta E\rangle}{\Delta t} = 2E\left(\frac{2\phi}{v} - \chi\right),$$

$$f_2 = \lim_{\Delta t\to 0}\frac{\langle(\Delta E)^2\rangle}{\Delta t} = \frac{8E^2\phi}{v},$$

where v is the electron velocity and $\chi = -\langle\Omega\rangle$ represents the reaction of the fast electron flux on the plasma. ϕ is given by

$$\phi = \tfrac{1}{2}\int_{-\infty}^{\infty} dz\langle\Omega(s,t)\Omega(s+z,t)\rangle.$$

Here s is the electron coordinate along the magnetic line at the instant t. At nonrelativistic

energies,

$$\lim_{\Delta t\to 0}\frac{\langle \Delta v\rangle}{\Delta t}=\phi-\chi v,$$

the mean acceleration. The parameters f_1, f_2 determine the Fokker-Planck equation for the distribution of E. However, we have neglected collisions between electrons. These may be taken into account by adding to f_1 and f_2 the correspondingly defined parameters arising from collisions. These are well known[6] and, in the E range where both contributions are comparable, we may write

$$f_1=2E[2(\phi/v)-\chi]-Z/E^{1/2},$$
$$f_2=8E^2\phi/v+2ZkT/E^{1/2},$$
$$Z=4\pi e^4 n\ln\Lambda/(2m)^{1/2}, \qquad (1)$$

where Λ is the Debye shielding parameter. At higher energies the effect of collisions may be neglected in comparison with Fermi acceleration; on the other hand, at smaller energies the values of f_1, f_2 given by (1) are incorrect; nevertheless they may be used in computing the steady-state solution of the Fokker-Planck equation since they give the correct (Boltzmann) distribution in this range. The Fokker-Planck equation,

$$\frac{\partial F(E,t)}{\partial t}=\frac{\partial^2}{\partial E^2}(\tfrac{1}{2}f_2F)-\frac{\partial}{\partial E}(f_1F),$$

has the steady solution

$$F(E)\propto f_2^{-1}\exp\int(2f_1/f_2)dE.$$

The integral can be evaluated in overlapping sections, yielding the approximately normalized result

$$F(E)=\frac{2}{\pi^{1/2}kT}\left(\frac{E}{kT}\right)^{1/2}\frac{[1+(\gamma E)^2]^{1/2}}{1+(\gamma E)^2W/mc^2}\exp\left[-\frac{\tan^{-1}\gamma E}{\gamma kT}\right]$$
$$\times\left[\frac{W}{mc^2}\left(1+\frac{v}{c}\right)\right]^{-\chi c/\phi};$$
$$\gamma=(2\phi/ZkT)^{1/2}(2m)^{1/4}. \qquad (2)$$

The nonrelativistic approximation to this spectrum, in the range $\gamma^{-1}\ll E\ll mc^2$, yields the exponential velocity distribution

$$G(v)=2(2m/\pi kT)^{1/2}(\gamma kT)^{-1}e^{-(\pi/2\gamma kT)}e^{-\chi v/\phi}. \qquad (3)$$

This may be used to estimate the mean energy flux density per electron, since the contribution of relativistic electrons is not a major one:

$$\frac{J}{n}\sim\int_0^\infty dv\,v\tfrac{1}{2}mv^2G(v)$$
$$=\frac{3}{\gamma}\left(\frac{2}{\pi}\right)^{1/2}nc\left(\frac{mc^2}{kT}\right)^{3/2}e^{-(\pi/2\gamma kT)}\left(\frac{\phi}{\chi c}\right)^4.$$

At not too great geocentric distances, the electron density n of the hydrogen plasma left behind by the departing storm may be obtained by writing

$$nMU^2/2\sim B^2/8\pi, \qquad (4)$$

where M is the proton mass. If the turbulent velocity is equated to the storm velocity, taken to be 500 km/sec, then, in the center of the outer belt where $B\sim 10^{-2}$ gauss, we obtain $n=2\times10^3$ electrons/cm^3. The effective electron temperature left behind by the storm is obtained by writing

$$\tfrac{3}{2}kT=\tfrac{1}{2}mU^2,$$

yielding $kT\sim 0.5$ ev. Thus Z can be evaluated. ϕ was previously estimated to be 2.5×10^5 cm/sec^2. Substituting this value in the expression for γ, we find $\gamma^{-1}\sim 10$ ev. This value is of course not reliable within at least a factor of 2. Within this range the observed value of J (2×10^{11} ev/cm^2 sec sterad)[7] can readily be obtained for any reasonable value (between 1 and 10) of the numerical parameter $\chi c/\phi$.

The presence of a gap between the two zones appears to be well accounted for by the sweeping action of the storm, regardless of whether the outer Van Allen radiation is brought from the sun or locally accelerated: the region just below the level to which the loops penetrate will be depleted of radiation by a Fermi acceleration, occurring along the geomagnetic lines, which dumps the trapped particles upon the earth. This region may be identified with the observed gap. The decrease in intensity at large geocentric distances, on the other hand, may be due to a great increase in the reaction parameter χ, as one goes farther out. In such regions, where the density is low and collisions relatively unimportant, all the electrons would like to be accelerated to high energies. Since the energy density available is much too small for this to happen, it appears that a large reaction parameter must be built up to prevent it. This will make the high-energy spectrum very steep. Thus it seems that, in order to produce electron radiation locally in the outer Van Allen belt, a Fermi acceleration is needed which is neither too weak nor too strong. The breakdown of the longitudinal invariant of the

electron's motion around a loop may also fail at large geocentric distances, with a consequent sharp reduction of the Fermi acceleration and therefore of the intensity.

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ION TEMPERATURE IN SCYLLA, AS DETERMINED FROM THE REACTION D(*d*,*p*)T*

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Scylla, the Los Alamos experiment on fast magnetic compression of a plasma, has been shown to produce about 10^7 neutrons per pulse from the reaction D(*d*,*n*)He^3.[1] An equivalent ion temperature of about 1 kev corresponding to this yield can be computed, using our measured values of the plasma density (6×10^{16}/cm^3), volume of the emitting region (3 cm^3), and mean duration of the neutron pulse (0.9 μsec). As has often been remarked, however, temperatures based on the reaction rate alone deserve little credence until the rate is so high as to be inexplicable on any other assumption than the thermal one. The present experiment was designed to measure in a more direct way the ion temperature. By studying the velocity spectrum of products of the reaction D(*d*,*p*)T we measure the mean square velocity of the center of mass of the *d*-*d* system, and hence the temperature of the plasma. We find in this way $T = 1.3$ kev. The spectrum obtained, coupled with the result that the neutrons emitted in the radial direction showed no Doppler shift on reversing the applied field,[1] argues strongly against many of the mechanisms for spurious neutron production which have been proposed. Radial hydromagnetic shocks having speeds of 15 cm/μsec are observed at the time of zero external magnetic field. It is easy to account for the measured final ion temperature with a model in which the plasma is adiabatically compressed after being preheated by the shocks. The assumption that the plasma is completely diamagnetic leads to the numerical result that the particle pressure is in balance with the confining magnetic field.

For a plasma of temperature T, the specific reaction rate is $\langle\sigma v\rangle_{\mathrm{Av}}$, namely the reaction cross section times the relative velocity averaged over a Maxwell distribution. The velocity spectrum of the protons depends on v_0, the minimum velocity of emission of a proton in the center-of-mass system, on kT, on the particle masses m_p, m_d, and m_t, and on the Q value of the reaction. We derive for the number of protons per unit energy interval

$$dN/dE = \left(\frac{m_d}{4\pi kT}\right)^{1/2} (n_d{}^2\langle\sigma v\rangle_{\mathrm{Av}}/m_p v_0) \times \exp\left\{-\left[\frac{m_d}{kT}(v-\bar{v})^2\right]\right\},$$

$$\bar{v} = v_0\left(1+\frac{kT}{2Q}\frac{d\ln\langle\sigma v\rangle_{\mathrm{Av}}}{d\ln kT}\right)$$

For protons

$$m_p v_0{}^2/2 = m_t Q/(m_p+m_t),$$

and for tritons the subscripts p and t are interchanged. By measuring the width of the velocity distribution of the products we obtain a value for T.

The axial direction was chosen because no windows in the discharge tube are required, and because the interpretation is simpler. The rest of the geometry was fixed by the parameters of a magnetic spectrograph which was available. Figure 1 shows in cross section the experimental

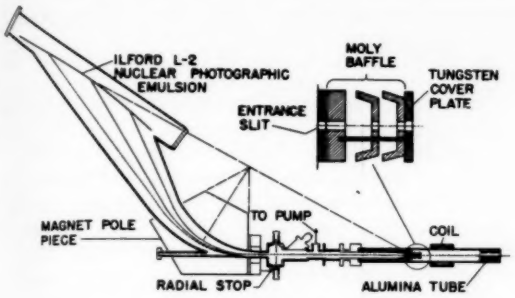

FIG. 1. Experimental arrangement. The discharge occurs within the alumina tube at the center of the coil. The dashed lines represent trajectories of the protons and the tritons from the *d–d* reaction.

arrangement. The Scylla discharge takes place inside the alumina tube. The spectrograph entrance slit is placed as close as possible to the discharge, namely 10 cm. The slit, if too near, is damaged by heat and wall material which strike it during late stages of the discharge. The slit forms the object for the ion-optical system, and the dispersed image is formed on the nuclear emulsion.

Two runs were made with the spectrograph and charged particles from Scylla. A thin cover was used to protect the emulsion from scattered light. In the first exposure the cover was 1.0 mg/cm² of (aluminized) polyester film; this effectively stopped the tritons, but left the protons with about 67 microns residual range. In the second exposure the cover was 0.55 mg/cm² of nickel; this allowed the tritons to enter the emulsion with a residual range of about 5 μ. The first exposure was 1617 discharges of Scylla, the second 1888, and the corresponding total numbers of neutrons emitted, as measured by a silver counter, were 1.0×10^{10} and 1.1×10^{10}. The plates after development showed tracks which were easily identified as the *d-d* protons or tritons, respectively. The density of tracks when plotted vs position revealed smooth distributions whose maxima came at the predicted locations. The width of the curve in the least favorable case was three times the resolution of the instrument and in the most favorable case eighteen times the resolution. The resolution and energy calibration were measured by means of an exposure to a thin source of Pu^{240} alpha particles. The track location fixes the particle velocity, and the number of tracks per unit length along the plate is converted into number of particles per energy interval coming

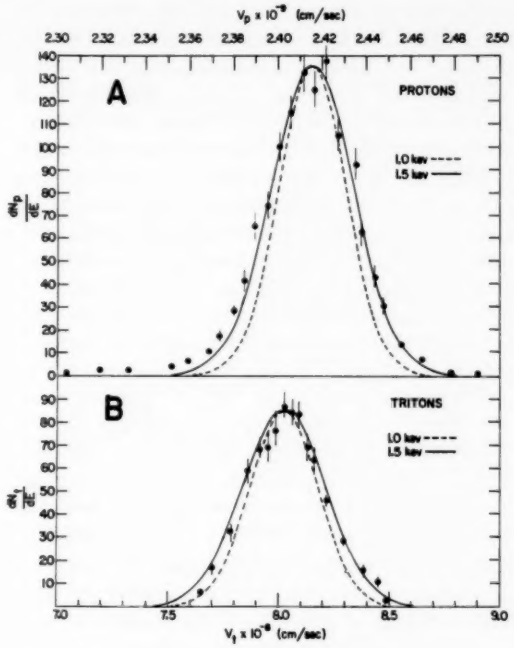

FIG. 2. Velocity spectra of protons and tritons from the *d–d* reaction in Scylla. The solid and the dashed curves are computed with the expressions given in the text. The width of the triton curve is eighteen times the measured resolution of the spectrograph.

from reactions in the plasma. The conversion takes into account the variations along the plate of angle of entry into the emulsion and of lateral magnification. Figure 2 shows the reduced distribution dN/dE for the second exposure. The curves are computed ones using the equation for dN/dE for $T = 1.0$ and $T = 1.5$ kev. A temperature of $T = 1.3$ kev is in agreement with these data and with similar data from the other exposure. The agreement of $\bar{v}$ with predicted values is as follows: for the protons 2.410×10^{9} cm/sec predicted, observed 2.412×10^{9} cm/sec, and for the tritons 0.8047×10^{9} cm/sec predicted, observed 0.8041×10^{9} cm/sec. This agreement indicates that the plasma has negligible mass motions in the axial direction. The principal uncertainty in the interpretation of the distributions is introduced by the low-energy tail to the proton distribution; however, this involves only about 1.5% of the reactions.

A more complete discussion of this experiment is being prepared for publication.

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TWO MASER EXPERIMENTS TO TEST GENERAL RELATIVITY

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Despite the fact that special relativity is firmly established on experimental grounds, the same cannot be claimed for the general theory. First of all, there are only three experimental tests of general relativity and not all of them are sufficiently conclusive in favor of the theory. (*i*) The bending of light rays, though correct as an order of magnitude, is just outside the experimental error. (*ii*) The observed shift of spectral lines towards red is qualitatively in agreement with the theory but a quantitative agreement is still a much discussed and investigated question. (*iii*) In the case of the advance of the perihelion of Mercury, the situation is different. Here, we have a satisfactory quantitative confirmation. Until recently, it was thought that the advance of the perihelion of Mars was not correctly predicted by the theory.[1] In the last few years, more elaborate calculations have shown that the theory predicts the effect within the observational error.[2] The calculations for the Earth's perihelion are at present in progress, but preliminary calculations are in agreement with the theory. It must be remembered that the perihelion motion is a second order effect and its quantitative agreement lends great support to the theory. Nevertheless, one cannot say that these facts provide sufficient tests for such a fundamental physical theory as general relativity. There is definite need for further practicable experiments in the field of gravitation. This need is being felt all the more strongly as time goes on because of man's increasing interest in space and gravitation.

In this note, two closely related but essentially different experimental tests of the theory are discussed. Only a year ago, it would have been impossible to carry them out. Thanks to the recent advances in maser techniques, these experiments can now be done without too much difficulty.[3]

The first experiment[4] provides a direct test for the principle of equivalence. As is well known, according to the principle of equivalence, the effects of an external gravitational field in a local coordinate system (say a small material box) can be eliminated completely by letting the coordinate system be free in that gravitational field (freely falling box). In such a coordinate system, the external gravitational field does not have any influence either on the motions of particles or on any other physical process whatever. Thus, according to the principle of equivalence, such a system must locally be equivalent to a Lorentz frame and in it, light rays must travel with the same velocity in all directions. Now, in the gravitational field of the Sun, our Earth may be considered as a small box and the above statement concerning the local propagation of light may be tested by comparing the velocity of light in the direction of the line joining the Earth and the Sun and in a direction perpendicular to it and the Earth's radius (Fig. 1). Nowadays, maser techniques are approaching an accuracy $\delta c/c \simeq 10^{-12}$ (c is the velocity of light) in comparing the two velocities.[4] If a discrepancy is found to this accuracy, it would imply a deviation from the principle of equivalence in first

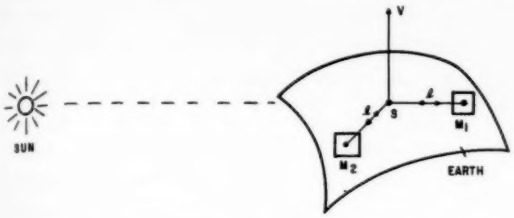

FIG. 1. SV is vertical; SM_1 and SM_2 are horizontal. M_1 and M_2 are mirrors and S is the light source.

order of the dimensionless quantity $GM/c^2R \simeq 9\times10^{-9}$, where M is the mass of the Sun, G the gravitational constant, and R is the distance of the Earth from the Sun. This dimensionless quantity is the expansion parameter involving the Sun's gravitational field. A higher order deviation would be presently undetectable.

The second experiment is similar in appearance but essentially different in nature. To see its function clearly, let us assume that the first experiment gave a negative result so that the principle of equivalence is confirmed. We can then ask the following question: Is the propagation of light the same in all directions in a coordinate system fixed in a gravitational field? A laboratory room on our Earth is such a fixed coordinate system in the gravitational field of the Earth. This question can be answered by comparing the velocity of light in the vertical and horizontal directions on the Earth (Fig. 2). If there is a first order influence due to the Earth's gravitational field, it would be of the

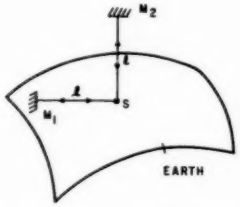

FIG. 2. SM_1 is horizontal and SM_2 is vertical. M_1 and M_2 are mirrors and S is the light source.

order of the dimensionless quantity $Gm/c^2r \simeq 7\times10^{-10}$, where m is the mass of the Earth and r its radius. This experiment does not test the principle of equivalence. It tests an implicit assumption, namely, the local isotropy of the space-time continuum. This assumption may conveniently be expressed as follows: The gravitational field influences all lengths and length-measuring rods in the same way. Also, all periodic phenomena and time measuring devices are influenced in the same proportion. As a result, one never is able to detect locally the effects of a gravitational field by kinematical means.

The second experiment is considerably more difficult then the first one because when one rotates the maser into the vertical directions, stresses and strains are introduced due to the Earth's gravitational field. In the first experiment, such complications can be avoided by doing the experiment during the sunrise or sunset. In this case, rotations are only around the vertical axis.[5]

In conclusion, we would like to emphasize that the experiments discussed here are essentially independent of any theory although it would have been difficult to present them without the aid of the general theory of relativity of Einstein.

The author would like to acknowledge a discussion with Professor C. H. Townes who made many essential suggestions.

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*On leave from the Applied Research Laboratory of Sylvania Electronic Systems, Waltham, Massachusetts.

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[5]Suggested by Professor C. H. Townes.

KNIGHT SHIFT IN SUPERCONDUCTORS*

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Although the theory proposed by Bardeen, Cooper, and Schrieffer[1] has been strikingly successful in explaining most properties of superconductors, the measurements of electron spin susceptibilities are supposed to present it with a serious dilemma. The BCS theory introduces strong coupling between pairs of particles with opposite spin and momentum, and therefore tends to prevent the electron spin reorientation necessary for low-temperature paramagnetism. However, observations of the electron spin paramagnetism, by the nuclear hyperfine interaction[2,3] (the Knight shift) in superconducting samples small enough to permit magnetic field penetration, indicate a substantial spin susceptibility. At absolute zero, the spin susceptibility observed in tin and mercury is approximately three quarters of the free electron value.

The purpose of this Letter is to suggest a mechanism for illuminating the Knight shift within the BCS framework. This mechanism occurs in consequence of the relative magnitude at low temperatures of the typical magnetic field wavelength, $\lambda = 1/k$ (approximately 10^{-5} to 10^{-6} cm), and the typical coherence distance, ξ_0 (approximately 10^{-4} to 10^{-5} cm). As these numbers indicate, the magnetic field contains Fourier components with wavelengths small compared to the size of the bound pairs. These components tend to polarize the pairs and thereby eliminate their compensation of the free electron spin susceptibility. Indeed, as the typical magnetic field wavevector increases, the paramagnetic spin susceptibility approaches the Pauli free electron value, $\chi^0 = 3n\mu^2/mv^2$. (In this expression μ and m are the electron moment and mass, n is the electron density, and v the Fermi velocity.)

In the first paper of a series on many-particle systems,[4] methods for determining thermal and electromagnetic properties of quantum mechanical systems have been developed. In a second, in preparation, these techniques will be shown to yield, for superconductors, the BCS solution with minor modifications which do not destroy the gap.[5] Using this approach, we have investigated the electromagnetic properties of superconductors, obtaining results which agree with the infrared absorption experiments on thin films,[6] and with the calculations of others.[7] We have also applied these methods to the electron spin paramagnetism of a superconductor in a static but spatially varying magnetic field, $\vec{H}$. In such a field, the spin magnetization, $\vec{M}$, is characterized by a nonlocal susceptibility, χ, which satisfies

$$\vec{M}(\vec{r}) = \int \chi(\vec{r} - \vec{r}')\vec{H}(\vec{r}')d\vec{r}'. \quad (1)$$

The Fourier transform of this spin susceptibility, $\chi(\vec{k})$, has been evaluated in terms of the attenuation parameter, $a = \frac{1}{2}\hbar vk/\Delta(\beta)$, the half energy gap, $\Delta(\beta)$, and the inverse of the absolute temperature in units of the energy, β. The result, derived by integrating the time-dependent correlation functions of the BCS model, may be expressed as

$$R(a,\beta) = 1 - \frac{\pi^2}{4a}\tanh(\tfrac{1}{2}\beta\Delta) + \frac{1}{2a}\int_0^\infty \frac{dx}{x}\ln\left|\frac{a+x}{a-x}\right| \frac{\tanh[\frac{1}{2}\beta\Delta(1+x^2)^{1/2}]}{(1+x^2)^{1/2}}, \quad (2)$$

where R is the ratio, $\chi(k)/\chi^0$.

In the large-attenuation or Pippard limit ($a \gg 1$), and at low temperatures, this ratio becomes

$$R(a \gg 1, \beta \to \infty) \approx 1 - \pi^2/4a = 1 - \pi\lambda/2\xi_0. \quad (3)$$

The experimental results are fit by taking $a \approx 10$ in (3). This value agrees qualitatively with estimates for a in bulk samples. (In bulk tin, for example, the coherence length is 2.5×10^{-5} cm and the penetration depth 5×10^{-6} cm, so that a is approximately eight.) A more quantitative comparison with (2) is hardly appropriate since its derivation employs the correlation functions of bulk material. In a detailed analysis the correlation functions of the small sample would have to be determined and the magnetic field variation then derived. (We note that if the boundary of the sample could be treated as incoherently scattering electrons, the effect of finite size would be more important than magnetic field variation in diminishing the correction to the free electron spin susceptibility. Specifically, if the size, or mean free path, l, were less than $\frac{3}{4}\pi\lambda$, R would become[8] $1 - \frac{3}{4}l/\xi_0$. While this treatment of the

boundary is known to be unsatisfactory,[9] and the use of bulk correlation functions is not justified, the argument underscores the possible magnitude of size-dependent corrections.)

In the small-attenuation or London limit ($a \ll 1$), the ratio (2) reduces to the form previously calculated by Yosida[10]:

$$R(0,\beta) = 2\int_0^\infty dy \frac{\exp[y^2+(\beta\Delta)^2]^{1/2}}{\{1+\exp[y^2+(\beta\Delta)^2]^{1/2}\}^2}. \qquad (4)$$

This limit, which predicts a rapidly decreasing Knight shift with decreasing temperature, is appropriate only very near the critical temperature, $1/\beta_c$, where it reduces to

$$R(0,\beta\approx\beta_c)\approx 1-0.60[\Delta(\beta)/\Delta(0)]^2. \qquad (5)$$

We are thus led by (2) to predict tentatively the following behavior for the spin susceptibility as the temperature is decreased: a sharp drop according to (5) immediately below the critical temperature, followed, perhaps, by a rise on entering the Pippard region, and then a smooth decrease to the asymptotic value (3). Observations of a similar structure have been reported by Knight,[11] but his preliminary data suggest that the sharp valley lies immediately above the critical temperature.

We therefore conclude that the Knight shift experiments are not at variance with the BCS theory. Furthermore, although the effects of size may be significant, the predictions for bulk samples agree qualitatively with the observations on small samples.

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†A. P. Sloane Foundation Fellow.

‡National Science Foundation predoctoral fellow.

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KNIGHT SHIFT IN SUPERCONDUCTORS

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(Received September 3, 1959)

The experimental values of the Knight shift found by Reif[1] on superconducting mercury colloids and by Androes and Knight[2] on evaporated tin plates when extrapolated to $T=0°K$ show that the electronic spin susceptibility $\chi_S(0)$ does not vanish for the superconducting ground state. These experiments are in disagreement with the calculation of Yosida[3] based on the BCS theory of superconductivity[4] which predicts an exponentially vanishing susceptibility near $T=0°K$ for a uniform magnetic field due to an energy gap $2\epsilon_0(0)$ for the creation of a pair of quasi-particles with total magnetic moment $2\mu_B$. Anderson[5] and Rickayzen[6] have generalized Yosida's calculation to include the effects of a space-varying magnetic field and they find a nonlocal susceptibility which is finite at $T=0$. Heine and Pippard[7] have suggested an alternative form for the matrix elements which enter the theory such that a finite Knight shift is obtained for a uniform field; however, no one has been able to construct wave functions which lead to these matrix elements.

A nonvanishing value of $\chi_S(0)$ can be obtained by including a mean free path, l, in constructing the normal state wave functions. Let the single-particle wave functions in the normal state, including the effects of the scattering centers which give rise to the finite mean free path, be ψ_n having energies ϵ_n. The Hamiltonian describing the interaction of the magnetic field H, which is taken to be in the z direction, and the electron spins is

$$\mathcal{H}_{\text{mag}} = \mu_B \sum_{nn'} \int \psi_{n'}^*(\vec{r}) H(\vec{r}) \psi_n(\vec{r}) d^3r \times \{c_{n'\uparrow}^\dagger c_{n\uparrow} - c_{-n\downarrow}^\dagger c_{-n'\downarrow}\}, \quad (1)$$

where $\psi_n = \psi_{-n}^*$ and the c's obey the usual Fermi anticommutation rules. If the BCS states are constructed in terms of the ψ_n's, the shift in energy to second order in H_{mag} when averaged over all positions of the scattering centers may be expressed as

$$\langle E^{(2)} \rangle_{\text{Av}} = -\mu_B^2 \langle \sum_{n,n'} \int \psi_{n'}^*(\vec{r}) H(\vec{r}) \psi_n(\vec{r}) \psi_n^*(\vec{r}') H(\vec{r}') \psi_{n'}(\vec{r}) d^3r\, d^3r'\, L(\epsilon_n, \epsilon_{n'}) \rangle_{\text{Av}}, \quad (2)$$

where $L(\epsilon_n, \epsilon_{n'})$ is given by (5.16) of I. In order to perform the sums in (2), we require averages of the form

$$\langle \sum_{\epsilon_n, \epsilon_{n'} = \text{const}} \psi_{n'}^*(\vec{r}) \psi_{n'}(\vec{r}') \psi_n^*(\vec{r}') \psi_n(\vec{r}) \rangle_{\text{Av}} = \frac{\sin kR \sin k'R}{kk'R^2} e^{-R/l}, \qquad R = |\vec{r} - \vec{r}'|, \quad (3)$$

where the average has been approximated by the product of the averages of the n and n' sums and we have used the results of Weiss and Abrahams[8] for the averages involved. The states ψ_n reduce to linear combinations of plane-wave states with wave number $|\vec{k}|$ in the limit $l \to \infty$.

The nonlocal susceptibility is given by

$$\chi(R) = \frac{-\delta^2 \langle E^{(2)} \rangle_{\text{Av}}}{\delta H(\vec{r}) \delta H(\vec{r}')} = \mu_B^2 \sum_{kk'} \frac{\sin kR \sin k'R}{kk'R^2} e^{-R/l} L(\epsilon_k, \epsilon_{k'}). \quad (4)$$

Since $L(\epsilon_k, \epsilon_{k'})$ is nonzero only when $|\epsilon_k|, |\epsilon_{k'}| \lesssim \epsilon_0$, it is easily seen from (5.3) and (5.15) of I that $\chi(R)$ is related to the kernel $K(R)$ for orbital diamagnetism by

$$\chi(R) = \chi_n [\delta(R) - \lambda_L^2(0) K(R)], \quad (5)$$

where $\chi_n = 2\mu_B^2 N(0)$ is the spin susceptibility in the normal state and $K(R)$ is given by

$$K(R) = \frac{J(R,T) e^{-R/l}}{4\pi \lambda_L^2(T) \xi_0}, \qquad J(R,T) \simeq e^{-R/\xi_0}. \quad (6)$$

Thus, for magnetic fields which vary slowly over distances of the order of ξ, we have

$$M(\vec{r}) = \int \chi(R) H(\vec{r}') d^3r' = \chi_n H(\vec{r}) \left\{ 1 - \frac{\xi}{\xi_0} \left[\frac{\lambda_L(0)}{\lambda_L(T)} \right]^2 \right\}, \quad (7)$$

where

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{l}.$$

For $l \to \infty$ this expression reduces to the result of Yosida; however, for finite l one obtains a nonvanishing magnetization at $T = 0$. To obtain agreement with the experiments of Reif and Androes and Knight one would have to choose l/ξ_0 to be 0.50 and 0.37, respectively. The coherence length, ξ_0, for tin is 2.5×10^{-5} cm,[4] so that $l \sim 10^{-5}$ cm for the latter experiment. This value is an order of magnitude larger than the mean dimension of Androes and Knight's plates, which suggests that a large amount of specular scattering occurs at the surface. The same situation exists for Reif's experiment.

It is possible that in small superconducting samples the low-lying continuum of magnetic states which gives rise to the Knight shift can be described by a continuous change of pairing with magnetization. For example, one could start with the magnetized state in the normal metal and begin pairing states on the spin-up Fermi surface with those on the spin-down Fermi surface in such a manner that a maximum number of pair states are connected by the two-body interaction, consistent with the conservation laws of the problem. If these states being paired were strictly plane waves, then momentum con-

servation would forbid the states interacting with each other, since in general the pairs would have different total momentum. However, the experiments mentioned above have been performed on samples whose dimensions were such that the spread in momentum of the single-particle states due to finite sample size was large compared to the momentum difference Δk due to the change in pairing caused by the magnetic field. Indeed, $\Delta k\, \xi_0 \ll 1$ for the magnetic fields employed in these experiments, where ξ_0 is the coherence length. It would appear that pairing exactly time-reversed states as one does for the ground state, or pairing states which differ slightly from these, should not alter the interaction energy strongly and it is possible that one would obtain a finite Knight shift in this manner. If this description were valid, then the magnetic continuum would be analogous to the continuum of current carrying states found by pairing single-particle plane-wave states $(\vec{k}+\vec{q}\uparrow, -\vec{k}+\vec{q}\downarrow)$. The vanishing of the nuclear spin relaxation[9] rate as $T \to 0$ indicates that the density of these magnetic states would be small just as in the case of the current-carrying states and that an energy gap would continue to exist for single-particle-like excitations.

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KNIGHT SHIFT IN SUPERCONDUCTORS

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Ferrell[1] has recently explained the observation by Reif[2] and by Androes and Knight[3] of finite Knight shifts in tin and mercury in terms of spin-reversing scattering (a spin-orbit effect) at the surfaces of the fine particles used. This theory might be questioned because it uses concepts—coherence length and plane wave states—which are well defined only for bulk samples of pure superconductors, while the experiments are done on fine particles. This Letter demonstrates that the spin susceptibility and Knight shift may be calculated using the theory of very imperfect ("dirty") superconductors proposed by the author,[4] with no further assumptions. The result confirms Ferrell's physical reasoning.

In a very imperfect metal, plane-wave states with fixed spin are no longer good one-electron functions, so the theory of reference 4 introduces hypothetical exact one-electron functions,

$$\psi_n = \sum_{\vec{k},\sigma} (n|\vec{k}\sigma)\psi_{\vec{k}\sigma}; \quad \text{energy } \epsilon_n . \qquad (1)$$

It is observed that in the absence of magnetic centers the Kramers time-reversal degeneracy must be present, so that the distinct state

$$\psi_{-n} = \sum_{\vec{k},\sigma} (n|\vec{k}\sigma)^* \psi_{-\vec{k}-\sigma},$$

is also an eigenstate of energy ϵ_n. Then it is shown that the pairing $(n, -n)$ leads to a BCS state with essentially the average energy gap of the bulk superconductor, explaining the fact that such samples as those of Androes and Knight have nearly the bulk T_c.

In the presence of spin-orbit scattering, the state ψ_n must not be an eigenstate of the spin; in fact it will normally have no average spin component in any direction. Thus the spin operator $\vec{S}$ has now only off-diagonal components, causing transitions from states n to n'. If ϵ_n and $\epsilon_{n'}$ usually differ by more than the gap, clearly the resulting spin susceptibility is not much affected by superconductivity; our theory merely expresses this physical fact.

Using time-reversal symmetry and Hermiticity, it may be shown that in terms of scattered function fermion operators c_n, the spin operator

S_z is

$$S_z = \tfrac{1}{2}\hbar \sum_{n,n'} S_{nn'}(c_n^* c_{n'} - c_{-n'}^* c_{-n}), \qquad (2)$$

where in terms of $\vec{k}, \sigma$ states it was

$$S_z = \tfrac{1}{2}\hbar \sum_{\vec{k}} (c_{\vec{k}\uparrow}^* c_{\vec{k}\uparrow} - c_{-\vec{k}\downarrow}^* c_{-\vec{k}\downarrow}).$$

The energy spectrum of the coefficient $S_{nn'}$ is in principle easily determined from a measurement of the absorptive component of the susceptibility in the normal state:

$$\chi''(\omega) \propto \sum_{n,n'} \delta(\epsilon_n - \epsilon_{n'} - \omega) |S_{nn'}|^2 = f(\omega\tau), \qquad (3)$$

defining the relaxation function f and the spin relaxation time τ. f might be taken proportional to $(1+\omega^2\tau^2)^{-1}$, for instance.

We actually use (3) in such a way that the average over n is taken only over functions with the same energy. This is no limitation; there is no reason to expect relaxation times to vary over ~1°K from the Fermi surface. It is this observation that one can use an observed relaxation function to get the energy dependence of the matrix elements that makes the technique work. A similar trick should give accurate results for the electrodynamics of dirty superconductors.

The susceptibility is to be calculated from

$$\chi = (\beta^2/\hbar^2)\textstyle\sum_m (0|S_z|m)(m|S_z|0)(E_m - E_0)^{-1}. \qquad (4)$$

If we assume a Bogolyubov[5] ground state defined by the "quasi-particle operators"

$$\alpha_n = u_n c_n^* - v_n c_{-n},$$

$$\alpha_{-n} = u_n c_{-n}^* + v_n c_n, \qquad (5)$$

(u_n and v_n are the usual functions of energy ϵ_n), only the part of S_z proportional to $\alpha_n^\dagger \alpha_{n'}^\dagger$ has matrix elements with the ground state; inserting (5) into (2) we get

$$S_z = \tfrac{1}{2}\hbar \sum_{n,n'} (u_n v_{n'} - u_{n'} v_n) \alpha_n^\dagger \alpha_{-n'}^\dagger S_{nn'} + \text{c.c.} + \alpha^\dagger\alpha \text{ terms}, \qquad (6)$$

and χ from (4) becomes

$$\chi = 4\beta^2 \sum_{n,n'} |S_{nn'}|^2 (u_n v_{n'} - u_{n'} v_n)^2 (E_n + E_{n'})^{-1}. \qquad (7)$$

Here $E_n = (\epsilon_n^2 + \epsilon_0^2)^{1/2}$. Clearly this is zero if $\epsilon_n = \epsilon_{n'}$, but otherwise not.

Since u, v, and ϵ are only functions of ϵ_n, we may use (3) to obtain the final answer in terms of the energy gap and relaxation function:

$$\frac{\chi_s}{\chi_n} = \frac{\int d\epsilon \int d\epsilon' f[(\epsilon - \epsilon')\tau\hbar^{-1}][u(\epsilon)v(\epsilon') - u(\epsilon')v(\epsilon)]^2 (E + E')^{-1}}{\lim_{\epsilon_0 \to 0} \int d\epsilon \int d\epsilon' \cdots} \qquad (8)$$

The quadrature could be done numerically; rough approximate results are

$$\chi_s/\chi_n \simeq 1 - (2\epsilon_0\tau/\hbar), \quad \epsilon_0 \ll h/\tau \qquad (9)$$

$$\chi_s/\chi_n \simeq \tfrac{1}{6}\langle(\epsilon - \epsilon')^2\rangle/\epsilon_0^2, \quad \epsilon_0 \gg h/\tau$$

$$\simeq \tfrac{1}{6}\hbar/\epsilon_0\tau. \qquad (10)$$

In the case of Knight and Androes' measurements on Sn, (9) gives $2\epsilon_0\tau/\hbar \simeq \frac{1}{5}$, and using the usual energy gap[6] this means $\tau \simeq 1.2\times10^{-13}$, corresponding to spin reversal once every 10-20 collisions with the surface: slightly less often than Ferrell found.

It is interesting that a finite Knight shift will be observed for almost any size of sample. Once the sample is large enough that the above effect does not work, the nonlocal nature of the electron susceptibility[7] will begin to be felt, so that a positive Knight shift will be seen in the surface layer, a negative one in the inaccessible interior. This will greatly complicate the size dependence.

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CYCLOTRON RESONANCE IN METALS*

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Well-resolved Azbel'-Kaner resonances (sometimes showing as many as $n=15$ subharmonics) have been observed in several metals (tin,[1] Al,[2,3] Zn[4]) including Cu.[5] With $\vec{H}$ normal to the surface of the metal, "dielectric anomalies" have been observed[4] even in the extreme anomalous limit. The characteristic feature of these effects is that they are more striking than predicted by theory.[6-8] The purpose of this note is to elaborate on an idea proposed by Chambers[7] and Heine[8] to explain the results obtained for $\vec{H}$ nearly parallel to the surface of the metal and to extend this idea to give a qualitative explanation of the masses observed with $\vec{H}$ normal to the surface of the metal.

With warped energy surfaces the electronic motion in the presence of a magnetic field can be discussed, according to Shockley,[9] in terms of "tubes" in k-space. A tube is a region in momentum space lying between k_H and k_H+dk_H, and E and $E+dE$; for degenerate statistics the effective tubes all lie at the Fermi surface and can be indexed by k_H. We can associate with each tube a drift velocity $\vec{v}_D$ defined as

$$\vec{v}_D(k_H)=\frac{1}{\hbar T}\int_0^T(\vec{\nabla}_k E)_{k_H,E_f}dt,$$

where the integration is taken over a complete period. From the equation of motion,

$$\hbar\frac{d\vec{k}}{dt}=\frac{e}{c}\frac{d\vec{r}}{dt}\times\vec{H},$$

it can be shown that $\vec{v}_D\parallel\vec{H}$.

Most treatments of this problem have assumed that the relaxation time τ is independent of k_H. The surface is then regarded as a boundary condition. An alternative formulation which helps in interpreting line shapes is obtained by making the substitution

$$\frac{1}{\tau}\rightarrow\frac{1}{\tau}+\frac{v_{Dn}(k_H)}{\beta\delta}. \qquad (1)$$

Here v_{Dn} is the magnitude of the component of the drift velocity normal to the surface of the metal, δ is the skin depth, and β is a number of order unity, similar to the quantity introduced by Pippard[10] in his "ineffectiveness" treatment of the anomalous skin effect. From (1) it is clear that for $\delta\ll l$ only tubes with $v_{Dn}\cong 0$ will contribute appreciably to the surface impedance.

The most surprising feature of the observed Azbel'-Kaner resonances is the large number of subharmonics that are resolved. These can be explained by assuming that the magnetic field makes an angle γ with the surface, where $\gamma\gtrsim\delta/r_c\cong 2\times10^{-7}H$ (for Cu; here r_c is the cyclotron radius, and $H\sim10^3-10^4$ gauss). In this case, as Chambers[7] and Heine[8] have indicated, $v_{Dn}\cong\gamma|\vec{v}_D|$ and only tubes with $\vec{v}_D\cong 0$ contribute to the resonance. In Cu with $\tau=10^{-9}$ sec, $\delta=10^{-5}$ cm, $v_F=10^8$ cm/sec, and $\gamma=10^{-2}$, the second term in (1) will be as large as the first for $v_D\cong 10^{-2}v_F$. Thus only narrow tubes contribute to the resonance, which accounts for the observation of unique masses.

If a particular Fermi surface is centrosymmetric, then $v_D(-k_H)=-v_D(k_H)$. In particular $v_D(0)=0$. The $k_H=0$ tube is also an "extremal mass" tube:

$$\omega_c(k_H)=\omega_c(0)+b^2k_H^2+\ldots, \qquad (2)$$

and this tube should contribute especially strongly to cyclotron resonances. We also have

$$v_D(k_H)=ck_H+\ldots \qquad (3)$$

In general conditions similar to (2) and (3) will not hold simultaneously for tubes other than $k_H=0$. Similar results may obtain, however, for orbits about especially symmetric points of Brillouin zones such as the centers of the (111) faces in fcc lattices. In these cases k_H in (2) and (3) should be replaced by $(k_H-k_H^0)$, where k_H^0 refers to the symmetry point in question. Whether such orbits are possible depends on the orientation of $\vec{H}$ relative to the crystal axes, and these special orbits may account for the large-angle resonances observed in tin.[1]

These remarks suggest that the following features should be observed in Cu. For $\vec{H}$ parallel to [100] the resonance associated with the $k_H=0$ tube is observed; here (2) and (3) may contribute about equally towards a unique mass. As $\vec{H}$ is rotated in the plane of the surface, the $k_H=0$ tube will continue to resonate; this produces a "normal" branch in the plot of m^* vs orientation

of $\vec{H}$ in the plane. We expect that b will increase as $\vec{H}$ departs from the [100] axis; this would ordinarily mean fewer subharmonics would be resolved. The fact that more are resolved can be explained by assuming that c increases faster than b. If the Fermi surface overlaps the zone edge near the center of the (111) face, m^* of the normal branch should have singularities when $\vec{H}$ is near the [110] direction, since then open orbits are possible. The normal branch thus "maps out" the Fermi surface. In addition to the normal branch, several anomalous branches may appear associated with the (111) contact areas [$\vec{k}_H{}^0 = \pi a^{-1}(1,1,1)$].

Quite different line shapes are expected depending on whether $C = (\gamma c/\delta)^2$ or $B = b^2$ is larger. When more subharmonics are resolved, so that C is larger, the masses are more unique, so that the asymmetrical lines predicted by Azbel' and Kaner in the ellipsoidal case should be more in evidence. In particular the asymmetry should be preserved in the higher subharmonics by (3) whereas (2) leads to a progressively symmetrical broadening.

Chambers[11] has argued that in the extreme anomalous limit Z should be independent of $\vec{H}$ for $\vec{H}$ normal to the surface. In this geometry τ in (1) should be replaced by $\tau/(1+i\omega'\tau)$, where (for circular polarization) $\omega' = \omega - \omega_c$. From (1) and (3) it can be seen, in agreement with Chambers' conclusion, that the decrease in τ is exactly compensated by an increase in the number of effective carriers. Under these conditions, however, Galt et al.[4] have observed large effects in Zn very similar to those obtained under classical conditions in Bi.[12] It is possible that these effects result from higher-order corrections, such as the variation of $\vec{E}$ in the skin depth, which are neglected in the ineffectiveness treatment. In any case the observed masses should result from tubes with $\vec{v}_D \cong 0$. With $\omega\tau \cong 10^2$, τ replaced by $1/\omega$ and $\gamma \cong 1$, we find as in the transverse case $v_D \lesssim 10^{-2} v_F$. Thus the same masses should be observed as in the transverse case; this conclusion agrees qualitatively with the results of Galt et al.[4] in Zn.

I am indebted to Dr. D. N. Langenberg and Mr. T. W. Moore for permission to use their results in Cu before publication. I have benefited greatly from conversations with Professor C. Kittel, Professor A. F. Kip, Dr. W. Harrison, Professor M. H. Cohen, and Professor M. Tinkham.

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CYCLOTRON RESONANCE IN COPPER*

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We are reporting preliminary results of a study of cyclotron resonance in copper. Observation of cyclotron resonance in a sample of natural copper consisting of a small number of single crystals was first reported by Langenberg, Kip, and Rosenblum.[1] The present experiments were done with an artificially grown single crystal and, because of improved experimental techniques and a more satisfactory sample, have yielded significantly more detailed results. These are the first observations of a resolved cyclotron resonance in a monovalent metal.

The experiments were done at 4.2°K and at a microwave frequency of 24 kMc/sec using a con-

ventional microwave bridge. One plane surface of the coin-shaped sample was cut within two degrees of a (110) plane and electropolished. This surface formed one end wall of a TE_{011} cylindrical cavity. The sample and cavity were mounted in a magnetic field so that they could be rotated together about an axis normal to the sample surface, keeping the magnetic field in the plane of the sample. Experiments were done both with and without magnetic field modulation. The results obtained with the two methods were consistent. Most of the results presented here are based on the field modulation data because these data were generally better.

The experimental results without field modulation showed a general decrease in sample surface resistance with increasing magnetic field. This decrease was estimated to be approximately 30% from zero field to the maximum value of 16 kilo-oersteds.

Well-resolved resonances were observed for all orientations of the magnetic field in the (110) plane. Only single masses were observed when the magnetic field was parallel to the ⟨100⟩, ⟨110⟩, and ⟨111⟩ axes; however, for most arbitrary orientations, at least two masses were well resolved. For some orientations, the anisotropy is such that the mass continuity is lost when making changes in orientation as small as two degrees.

Cyclotron masses were calculated from the period $\Delta(1/H)$ of the derivative maxima, as shown in Fig. 1. The masses for H parallel to the principal crystal axes and their behavior with respect to orientation are given in Table I; the value of $m^*(111)$ is in excellent agreement with the value estimated by Shoenberg[2] from the temperature dependence of the de Haas–van Alphen effect. Although the observed masses lay between extreme limits of $0.5m_e$ and $5.0m_e$ (both values

Table I. Cyclotron masses for copper with H in (110) plane parallel to principal crystal axes.

Axis	Mass	Behavior of mass with respect to orientation
⟨100⟩	$(1.32 \pm 0.02)m_e$	maximum
⟨111⟩	$(1.30 \pm 0.03)m_e$	increasing as H rotated toward ⟨110⟩ axis
⟨110⟩	$(1.12 \pm 0.02)m_e$	minimum

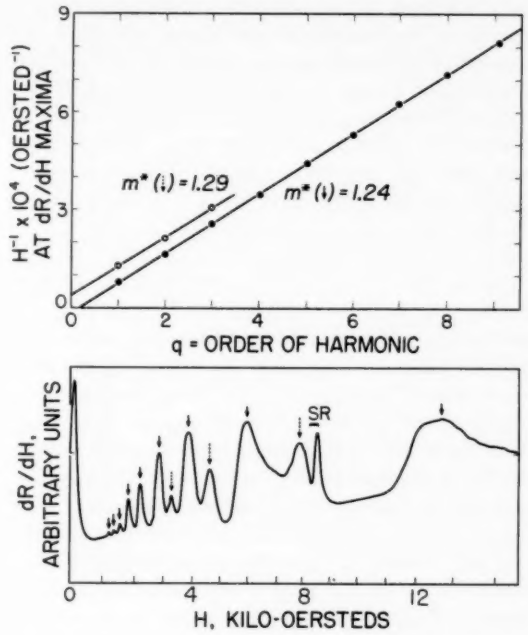

FIG. 1. The lower panel shows a recorder trace of absorption derivative for H in a (110) plane 10° from a ⟨100⟩ axis. J_{rf} is in a (110) plane and 45 degrees from H, 55 degrees from the ⟨100⟩ axis. $f = 24.47$ kMc/sec. No significance should be attached to the envelope of the derivative peaks since the lock-in detector phase varied with magnetic field. The upper panel is a plot of reciprocal magnetic fields at the absorption derivative maxima against integers; $m^* = e/[\omega c \Delta(1/H)]$, where $\Delta(1/H)$ is the slope of the $1/H$ plot. Note the large "phase" discrepancy between the two nearly equal masses.

observed at the same orientation, with H 40 degrees from a ⟨100⟩ axis), the masses for most orientations were between $1.1m_e$ and $1.4m_e$. Experimental errors in the masses themselves were usually less than 2%; the largest uncertainty in quoting masses for specific orientations where the anisotropy was large arises from small uncertainties in orientation.

For finite relaxation times (we estimate $\omega\tau \sim 50$ in the present case), small corrections should be applied to these experimental masses because absorption derivative maxima rather than absorption minima have been used and because the absorption minima themselves do not occur at exactly the cyclotron fields. No attempt has been made to apply these corrections since the latter one depends upon the "character" of the mass,

i.e., whether the associated carriers are on a region of the Fermi surface where the mass is a relative minimum or maximum, or ellipsoidal (Azbel' and Kaner[3]), and this is not yet known with any certainty. We believe the net mass correction to be small in any case, certainly less than 5%, since the two corrections tend to cancel one another, $\omega\tau$ is large, and there is no detectable curvature in the $1/H$ plots.

These resonances are perhaps the first in any metal for which the relationship with the Fermi surface appears fairly clear and from which further detailed information about the surface may be derived. Pippard,[4] on the basis of anomalous skin-effect experiments, has proposed a model of the Fermi surface of copper which lies entirely in the first Brillouin zone and is very nearly spherical except for "humps" which contact the hexagonal zone boundaries around $\langle 111 \rangle$ directions; Olson and Rodriguez[5] also conclude that the surface contacts the zone boundary, from an analysis of magnetoresistance measurements. The very complexity of our data seems to argue against the existence of a single, closed convex surface. If Pippard's model is at least qualitatively correct, the following features should be apparent in the resonances: for H parallel to a $\langle 100 \rangle$ axis, closed orbits are available to carriers having small momentum along the field, and those having no momentum along the field should display a resonance, since by symmetry they have an external mass. If the resonance due to these "equatorial" orbits is followed as H is rotated toward a $\langle 110 \rangle$ axis, the associated masses should change only slowly over the nearly spherical part of the Fermi surface, but should have singularities at the three orientations for which the "equatorial" orbits just intersect the edges of the contact areas. Our data are in excellent agreement with these comments. One of the expected mass singularities has been observed in detail with H $18° \pm 1°$ from a $\langle 110 \rangle$ axis, as compared to $14\frac{1}{2}°$ predicted by the Pippard model; the present data are not complete enough for us to observe the other two expected singularities.

For H along the $\langle 100 \rangle$ and $\langle 110 \rangle$ axes only 8 or 9 harmonics are resolved and the shape of the derivative peaks is fairly symmetrical, while for H along arbitrary axes 12 harmonics are often observed and the peaks have the asymmetrical "N" shape characteristic of the Azbel'-Kaner theory for ellipsoidal (unique) masses and high $\omega\tau$; in general, the peaks become increasingly symmetrical as the order of the harmonic increases. The implications of these results are considered by Phillips.[6]

An interesting unexplained feature of the resonance is that at some orientations the "phase" of the oscillations is not that predicted by Azbel' and Kaner; i.e., if the period of the oscillations is fitted to the Azbel'-Kaner curve, the positions of the peaks are shifted with respect to the theoretical curve. This phase shift is most easily observed by noting the intercept of the $1/H$ plots (see Fig. 1). In at least one case a continuous and rather rapid variation of the phase is observed as the magnetic field orientation is changed. A phase discrepancy has also been observed by Aubrey and Chambers[7] in bismuth and by Galt[8] et al. in zinc.

This work is being continued and a complete report will be published at a later date.

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THEORY OF THE RESISTANCE MINIMUM IN DILUTE PARAMAGNETIC ALLOYS

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The presence of a minimum in the electrical resistivity as a function of temperature, at low temperatures, appears to be a general property of dilute paramagnetic alloys of a transition metal in a noble metal.[1] Recent experiments[2] on Cu-Fe alloys indicate that the resistance minimum is a true bulk property depending on the presence of paramagnetic ions in random solid solution.

It has been shown that the scattering of conduction electrons by paramagnetic ions yields a temperature-independent contribution to the resistivity.[3] However, if the Zeeman levels of the ions are split by a magnetic field (for example), this contribution becomes temperature dependent. The anomalous resistivity of paramagnetic alloys suggests the presence of scattering centers with closely spaced energy states. Such centers occur when the interaction between nearest-neighbor pairs of paramagnetic ions is considered. If the spin of each ion is S, then states of the ion-pair are characterized by the total spin I, which has integer values between 0 and $2S$. The energy separation of these states of different I depends on an exchange integral W. We have calculated the resistance arising from scattering by such ion-pairs, and find that if the interaction of a pair is ferromagnetic ($W>0$), a resistance minimum should occur. Since this mechanism involves pairs of ions, the size of the minimum should be proportional to the square of the concentration of the transition metal. This quadratic dependence appears to be qualitatively confirmed for the most dilute Cu-Co alloys investigated by Jacobs and Schmitt.[4] [One expects this dependence to obtain only for extreme dilution, where the $(2I+1)$-fold degeneracy of each level is not split by other interactions.]

A mechanism for a temperature-dependent resistivity is described in the heuristic model of Schmitt.[5] The resistivity caused by elastic scattering will be temperature dependent since the occupancy of the different energy states, having different scattering cross sections, varies with temperature. Inelastic scattering is temperature dependent for the same reason and also because the available final states for the scattered electron depend on temperature.

We shall treat the s-d exchange scattering of conduction electrons by ferromagnetically coupled pairs. The model Hamiltonian is the sum of two terms:

$$H_0 = -W\vec{S}_1 \cdot \vec{S}_2, \tag{1}$$

and

$$H_1 = V(\vec{r}) + V(\vec{r}-\vec{R}) - 2J(\vec{r})\vec{s}\cdot\vec{S}_1 - 2J(\vec{r}-\vec{R})\vec{s}\cdot\vec{S}_2. \tag{2}$$

$\vec{S}_1$ and $\vec{S}_2$ are the spin operators for the ions, $\vec{s}$ the spin operator for an electron in the conduction band of the solvent metal, and $\vec{R}$ is the nearest-neighbor distance. $V(\vec{r})$ and $J(\vec{r})$ denote the spin-independent and spin-dependent coefficients of the interaction between a conduction electron and an ion. The eigenvalues of H_0 are given by

$$E_I = \tfrac{1}{2}W[I(I+1) - 2S(S+1)],$$

each level being $(2I+1)$-fold degenerate.

We find that the average elastic scattering cross section for <u>each</u> of the $2I+1$ states of energy E_I is proportional to $I(I+1)$. If $W>0$, states of lower energy have larger elastic scattering cross section, so that this contribution to the resistance increases as temperature decreases. Therefore a resistance minimum would appear to be easily explained. However, one must take account of inelastic scattering, which becomes frozen out at low temperatures and yields a decreasing contribution to the resistance as temperature decreases. The net temperature dependence depends on the close competition of elastic and inelastic processes, so one must explore the model in detail to show that the elastic contribution predominates.

The scattering arising from the V terms of (2) is temperature independent and need not be considered further. All cross terms between V and J terms may be shown to vanish when square matrix elements are averaged over conduction electron spin states. The J terms yield the following elastic and inelastic contributions to the resistivity:

$$\rho_{\text{el}} = \alpha f_+ B_+, \tag{3}$$

$$\rho_{\text{inel}} = \alpha f_- B_-, \tag{4}$$

where

$$\alpha = m^2 N_p / 2\pi n e^2 \hbar^2 K^3,$$

N_p being the number of pairs per cc, n the electron concentration, and K the Fermi wave number. The factors $B_\pm$ are the following integrals of the Fourier transform $J(q)$ of $J(\vec{r})$:

$$B_\pm = \int_0^{2K} |J(q)|^2 [1 \pm (\sin qR/qR)] q^3 dq. \qquad (5)$$

The second term in the square brackets of (5) represents the interference of the scattered waves from the two atoms of a pair (averaged over random directions of $\vec{R}$). One should observe that the interference is constructive for elastic scattering, whereas it is destructive for inelastic scattering. It is precisely this feature which allows the elastic scattering to predominate over the inelastic. The remaining factors, $f_\pm$, of (3) and (4) contain the temperature dependence of the resistivities and are

$$f_+ = \sum_{I=0}^{2S} p_I (2I+1) I(I+1)/4, \qquad (6)$$

$$f_- = \sum_{I=1}^{2S} p_I p_{I-1} I[(2S+1)^2 - I^2]/(p_I + p_{I-1}), \qquad (7)$$

where p_I is the equilibrium probability that each state of energy E_I is occupied.

Consider now the temperature dependence of the resistivity contributions. For $W/kT < 1$, Eqs. (6) and (7) become

$$f_\pm \cong \tfrac{1}{2} S(S+1)[1 \pm S(S+1) W/3kT].$$

Therefore, the net temperature-dependent contribution to the resistivity is

$$\Delta\rho = \alpha S^2 (S+1)^2 (B_+ - B_-) W/6kT. \qquad (8)$$

Since $B_+ > B_-$ [assuming only that $J(q)$ is well behaved], the resistivity will increase with decreasing temperature. One can show that this increase will be monotonic all the way to 0°K as long as

$$B_+/B_- > 2(4S+1)/(4S-1). \qquad (9)$$

If the inequality (9) is not satisfied, the resistivity will go through a maximum before reaching its 0°K value. In either case the competition between (8) and the phonon resistivity should produce a minimum.

The magnitude of the anomalous resistivity increase also agrees with experiment for reasonable values of J and W. A detailed account of this work will be published elsewhere.

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ATTENUATION OF SOUND IN A GERMANIUM CRYSTAL AT ULTRA-HIGH FREQUENCIES AND LOW TEMPERATURES*

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We have measured the ultrasonic attenuation of compressional and shear waves in a high-purity crystal of germanium at frequencies up to 650 Mc/sec and at temperatures down to 1.5°K. At room temperature there is some evidence for the dislocation loss mechanism, but at low temperatures the attenuation is very small.

Previous measurements of ultrasonic attenuation at high frequencies in germanium have been at room temperature, and limited to 90 Mc/sec in the guided-wave method[1] and to 300 Mc/sec in the usual, unbounded medium, method.[2] We have added an ultra-high-frequency pulsed oscillator to our equipment to extend the frequency range to 700 Mc/sec. The low temperatures were achieved in a conventional liquid helium cryostat, the electrical energy passing down a bifilar line to the transducer, which was bonded to the parallel-sided specimen.

The specimen was of n-type germanium, with room temperature resistivity 45 ohm-cm and net donor concentration 1×10^{12}/cc. Its flat faces

were oriented to be normal, within half a degree, to the [100] direction. They were both polished to a mirror finish and were parallel within 25×10^{-6} in./in.

The measured (or apparent) attenuation coefficient includes, in addition to the absorption in the specimen, a loss on reflection at the end faces[3,4] and a loss due to diffraction of the beam.[5] In the room temperature measurements we have applied a measured reflection correction[3] and a calculated diffraction correction[5] to both sets of results, giving the intrinsic attenuation curves of Fig. 1. For the compressional waves, the small thermoelastic loss[6] in the specimen has been subtracted, leaving only losses due to other intrinsic mechanisms.

The results for compressional waves are in agreement with those of Redwood at the lower frequencies and show an approximately square-law dependence on frequency—the measured slopes being 1.85 for the compressional waves and 1.90 for the shear waves. The dislocation-damping theory[7,2] predicts a square-law frequency dependence up to a frequency ω_m, above which the attenuation due to this mechanism should become nearly independent of frequency. The results at the highest frequencies seem to confirm the presence of a dislocation resonance, with ω_m about 500 Mc/sec, corresponding to a lower resonant frequency.

Most of this high-frequency attenuation disappears at liquid helium temperatures, however, as is shown in Fig. 2, for compressional waves at 340 and 500 Mc/sec and for shear waves at 333 Mc/sec. The apparent attenuation is independent of temperature below 15°K and in this region is so low that it may be largely due to the reflection losses, which we have not yet measured at low temperatures. The points shown above the dashed curve in Fig. 2 were observed on warming the sample, but not on cooling, and so may be due to changes in the bond. That a satisfactory high-frequency bond was achieved at helium temperatures is shown by the photograph (Fig. 3) of the compressional waves at 340 Mc/sec.

The rise in attenuation that begins at about 20°K is similar to that found in quartz by Bömmel and Dransfeld.[8] This rise suggests that the phonon-phonon scattering associated with Umklapp processes, first discovered in thermal conductivity measurements in dielectric crystals, may be present here. It is of interest to note that the phonon mean free path for Umklapp collisions in germanium, as estimated by the method of Berman et al.,[9] is about 12 microns at 20°K, or

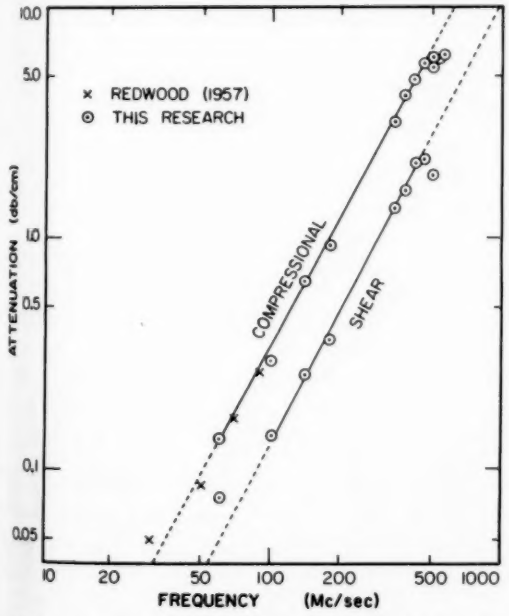

FIG. 1. Intrinsic attenuation of sound in high-purity germanium as a function of frequency at room temperature.

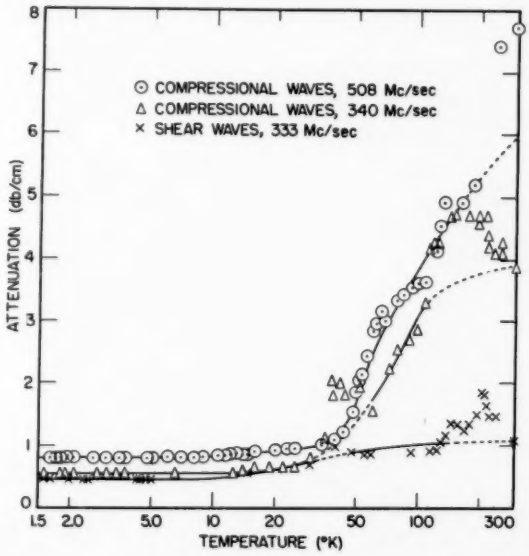

FIG. 2. Apparent attenuation of sound in high-purity germanium as a function of temperature. The dashed portions indicate regions where experimental effects, such as bond transformations, may be causing excess attenuation.

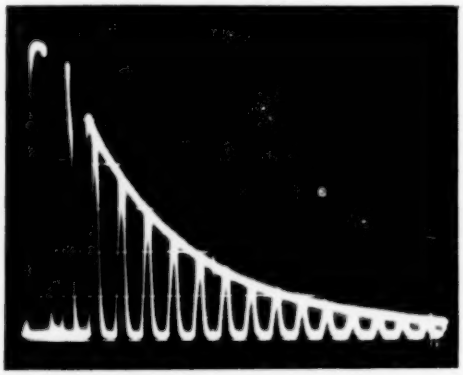

FIG. 3. Initial pulse and a series of echoes formed by 340-Mc/sec compressional waves in a germanium crystal at 2°K. The exponential curve is generated electronically and used in the measurement of the attenuation coefficient.

roughly the same as the acoustic wavelengths, which are 9.8, 14.5, and 10.4 microns for the waves shown in Fig. 2. Measurements at still higher frequencies are clearly possible at low temperatures and we are extending our results into the kilomegacycle region.

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†On leave from Queen Mary College, London, England.

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HOT ELECTRONS AND CARRIER MULTIPLICATION IN SILICON AT LOW TEMPERATURE

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Under equilibrium conditions the mean energy of the electrons in a semiconductor is of the order of kT_0, where T_0 is the lattice temperature. In recent years considerable experimental work has shown that the electron energy can be increased substantially above kT_0 when an electric field is applied to the sample.[1] The electrons are heated while the temperature of the lattice remains constant. For very high electric fields the collision frequency between electrons and lattice increases and limits the electron drift velocity. These effects were first observed by Ryder and Shockley in n-type Ge at fields of 10^3 v/cm.[2] The increased carrier mobility at low temperatures reduces the electric field necessary for the observation of hot carriers. At temperatures where most of the carriers are frozen out on impurity levels, new effects are observed when the electrons become energetic enough to ionize donor impurities. Carrier multiplication by this process has been studied in Ge in the temperature range of liquid He.[3]

We have measured the electrical resistivity and the Hall coefficient as a function of the electric field in phosphorus-doped silicon ($E_i = 0.044$ ev) at 20°K. The samples (usual size $10\times2\times2$ mm^3) were provided with n^+ contacts. The carrier concentration, n, and the Hall mobility, μ, were calculated from the relations $n = 1/R_H e$ and $\mu = R_H/\rho$, where $\rho = E/j$. In Figs. 1 and 2 the current density j, n, and μ are plotted as a function of electric field for two samples containing less than 10^{16} oxygen atoms per cm^3. The room temperature resistivities and the donor and acceptor concentrations are listed on the figures.

To simplify the discussion we divide our field range into four parts:

(1) At low electric fields, $E \lesssim 5$ v/cm, the sam-

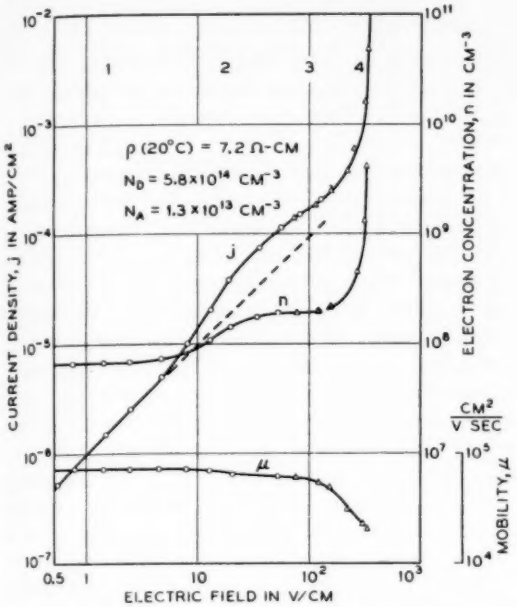

FIG. 1. Current density j, carrier concentration n, and Hall mobility μ versus electric field. (o dc, △ pulses.) $\rho = 7.2$ ohm cm.

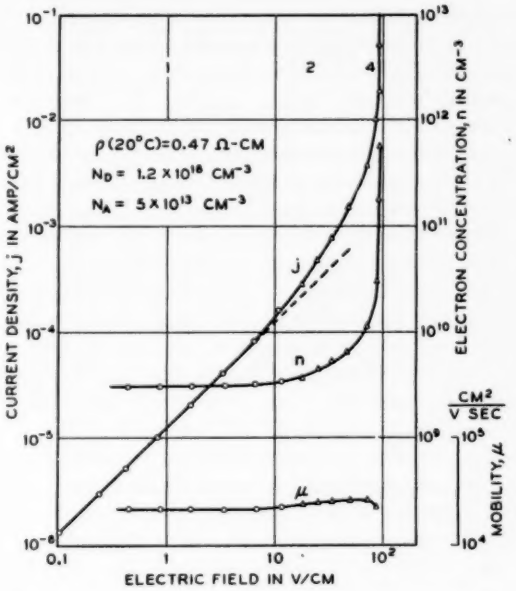

FIG. 2. Current density j, carrier concentration n, and Hall mobility μ versus electric field. (o dc, △ pulses.) $\rho = 0.47$ ohm cm.

ple is Ohmic, i.e., the mobility μ_0 and carrier concentration n_0 are independent of the applied electric field.

(2) Above ~10 v/cm, the carrier concentration increases (approximately by a factor of three in Fig. 1). The mobility increases in samples where μ_0 is affected by the scattering of ionized impurities (Fig. 2) and decreases slightly where μ_0 is mainly determined by lattice scattering (Fig. 1).

(3) When the electrons have attained a critical velocity[4] $v_c = \mu E = 2\times10^6$ cm/sec, the mobility decreases strongly in all samples investigated. The rate of carrier formation tends to decrease. This field range is limited by the onset of (4) (e.g., in Fig. 2, region 3 is hardly observable).

(4) At high fields, the carrier concentration increases rapidly, resulting in a sharp rise of the j-E characteristic. The following observations should be added: (a) The breakdown phenomenon is a true field effect, as was substantiated on samples between 1.1×10^{-2} cm and 1.2 cm of length. (b) The breakdown field E_B is determined by the bulk properties of the specimen. Variation (factor 3) of surface-to-volume ratio did not affect E_B. (c) E_B appeared to be independent of the contacts used. Diffused n^+ layers or alloyed (Au-Sb) contacts gave identical results on the same material. (d) A specimen was constructed which consisted of a large part (length 1 cm, cross section 2.9 cm^2) with a small needle extension on one end (length 0.2 cm, cross section 2.3×10^{-3} cm^2). Diffused contacts were provided to the bottom end of the large part (A), its junction with the needle (B), and to the end of the needle (C). Current was passed through contacts (A) and (C) while the potential could be measured between (A) and (B) or (B) and (C). Breakdown was observed in the needle while the large part of the crystal was still in the Ohmic range. The results were found to be independent of polarity.

Our observations can be explained as follows. Region (1): At low electric fields the electrons are in equilibrium with the lattice and the resistivity is constant. (Note that the contacts are Ohmic.) Region (2): The increase in electron concentration with electric field is considered to be the result of the decreasing thermal recombination rate (between free electrons and ionized donor states) with growing electron velocity. Experimental evidence for this effect was recently given by Koenig for n-type Ge in the temperature range of liquid helium.[5] In samples

where ionized impurity scattering dominates the carrier mobility, μ_0 increases with electric field since scattering of ionized impurities is less effective for hot electrons.[6] Region (3): Shockley's theory,[7] which considers the interaction of hot electrons with longitudinal acoustical phonons, predicts a critical drift velocity of $v_c = 1.5 \times c = 1.4 \times 10^6$ cm/sec,[8] in agreement with the observed value of 2×10^6 cm/sec. The measured mobility decrease does not follow the predicted $E^{-1/2}$ dependence, however. In n-type Ge ($T_0 = 300°K$) agreement between experiment and theory has been reported after consideration of interactions with optical phonons.[9] No theory has been advanced which takes into account the multivalley structure of n-type Si. Region (4): Several processes have to be considered which can give rise to a carrier increase in a semiconductor: Injection of carriers was excluded by large area n^+ contacts. The experiment discussed under 4(d) is believed to be conclusive evidence that carrier injection through contacts is not responsible for the observed breakdown phenomena. Field emission from the valence band or from the donor levels is ruled out since the probability for this process is too small for electric fields below 10^5 v/cm. Heating of the sample was avoided by immersing the specimen into liquid H_2 and by using short (square) voltage pulses. In the measurements shown in Fig. 1 and 2, the voltage and current pulses (5×10^{-3} sec duration; 1 pulse per sec) were found to be perfectly flat.[10] At higher power level the current increased during the application of the pulse due to a small heating of the specimen. For pulses of 5×10^{-6} sec duration (15 pulses per sec), however, the current remained constant up to current densities several decades larger than shown in the figures. The process most likely responsible for the sharp rise in carrier concentration is the ionization of donor impurities by hot electrons.[11] Preliminary experiments indicate that the breakdown voltage depends upon N_D and N_A, the concentration of the majority and minority impurities. Further investigations are necessary to elucidate this point.

Heating of holes and rapid increase in carrier concentration were also observed in boron-doped silicon ($E_i = 0.045$ ev). The effects are similar to those discussed above; v_c was found to be 1×10^6 cm/sec.

In a recent article, Lampert, Herman, and Steele[12] pointed out that in silicon the energy of the intervalley acoustical phonons is smaller than E_i, the impurity activation energy of phosphorus. They suggest that low-temperature breakdown will not occur, because the hot electrons lose a large fraction of their energy through emission of a single phonon before they reach the E_i value. The results reported here indicate that in spite of this loss mechanism enough electrons reach the threshold energy E_i and carrier multiplication can take place.[13] For p-type silicon (boron) the optical phonon energy ($E_{opt} = 0.063$ ev) is considerably larger than $E_i = 0.045$ ev and impact ionization is expected[12] in agreement with our observations.

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[1]For references see J. B. Gunn, Progress in Semiconductors (John Wiley and Sons, Inc., New York, 1957), Vol. 2; and S. H. Koenig, J. Phys. Chem. Solids 8, 227 (1959).

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[4]The assumption Hall mobility = drift mobility is made, which will be correct within 10% in n-type Si for magnetic fields of 2×10^3 gauss (C. Herring, private communication).

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[7]W. Shockley, Bell System Tech. J. 30, 990 (1950).

[8]$c = 9.2 \times 10^5$ cm/sec is the velocity of the longitudinal acoustical waves. McSkimin, Bond, Buehler, and Teal, Phys. Rev. 83, 1080 (1951).

[9]E. Conwell, J. Chem. Phys. Solids 8, 234 (1959).

[10]A closer inspection on the microsecond level shows some structure at the beginning of the pulse in the initial range of the breakdown characteristic.

[11]Hot electrons and breakdown resulting from impact ionization of donors were recently observed at 1.3°K by G. Feher, Phys. Rev. Letters 3, 135 (1959).

[12]Lambert, Herman, and Steele, Phys. Rev. Letters 2, 394 (1959).

[13]In the high electric fields of p-n junctions, electrons can gain energies of 1 to 2 ev, far above the energy of acoustical and optical phonons. Avalanche breakdown by this mechanism has been extensively investigated.[1]

APPARENT STRUCTURE ON THE FAR INFRARED ENERGY GAP IN SUPERCONDUCTING LEAD AND MERCURY*

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(Received September 8, 1959)

We have performed two different experiments which provide evidence for apparent structure on the edge of the energy gap in superconducting lead and mercury at low temperatures (~1.5°K). In one experiment, described in a previous Letter by two of us,[1] we measure the far infrared power reaching a bolometer after many reflections from the walls of a nonresonant cavity. The power is called P_S and P_N for the superconducting and normal states, respectively. Figure 1 shows the data for a lead and a mercury cavity. The sudden decrease in $(P_S - P_N)/P_N$ with increasing photon energy signifies the onset of absorption in the superconducting state due to the excitation of electrons across the energy gap. We believe that the dip in the curve at photon energies just below the main absorption edge represents a precursor absorption in the superconducting state, since the absorption in the normal state is expected to rise smoothly and monotonically with increasing frequency. This dip is evident in our previously published data[1] on lead. Repetition of the experiment has confirmed this feature in lead, and has shown an even more pronounced dip for mercury.

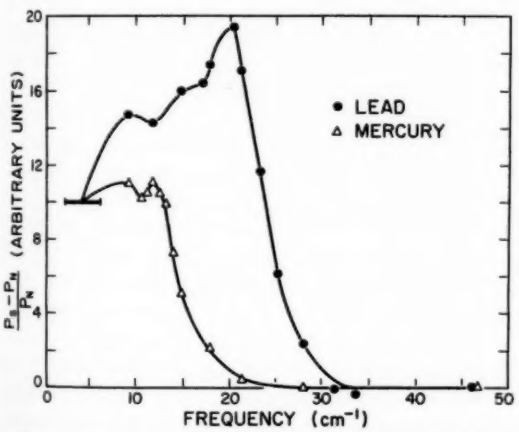

FIG. 1. Frequency dependence for mercury and lead of the fractional difference between the power reaching the bolometer in the superconducting and normal states. These curves have been normalized for display purposes so that the ordinate of the lowest frequency point is the same for each of them. The horizontal bar indicates the approximate bandwidth of the lowest frequency point. The bandwidth for the other points was ~10%.

The structure was actually first noticed in measurements by two of us (D.M.G. and M.T.) of far infrared transmission through films about 10 A thick. These experiments are similar to those performed on lead and tin by Glover and Tinkham,[2] but with accuracy improved by changes in optics and radiation detection. The magnitude of the ratio T_S/T_N of the transmission through the superconducting film to that through the normal film is measured as a function of photon energy. The data are analyzed, using the Kramers-Kronig relations and a conductivity sum rule,[3] to obtain the frequency dependence of $\sigma_1(\omega)/\sigma_N$ and $\sigma_2(\omega)/\sigma_N$, where σ_1 and σ_2 are the real and imaginary parts of the superconducting conductivity, and σ_N is the normal conductivity. The simplest models for an energy gap, of width $\hbar\omega_g$, predict that $\sigma_1(\omega)$ is zero for $\omega < \omega_g$, and that $\sigma_1(\omega)$ rises monotonically towards σ_N for $\omega > \omega_g$. Actually, there appears to be a preliminary hump in $\sigma_1(\omega)/\sigma_N$ for lead before the main absorption sets in, as shown in Fig. 2. The curve for a second lead film has a qualitatively similar frequency dependence. Unfortunately, the mercury films have not been reproducible enough to permit a meaningful calculation of the conductivity ratio. However, a dip in the transmission ratio, corresponding to a hump in $\sigma_1(\omega)/\sigma_N$, is clearly evident in the transmission curves of two different

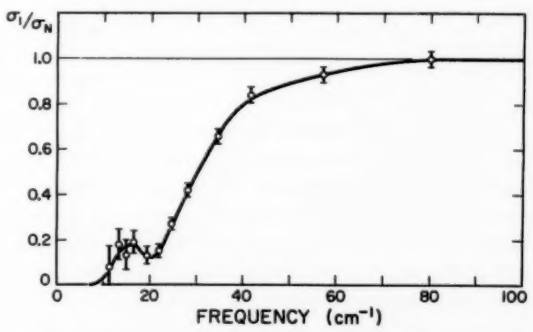

FIG. 2. Frequency dependence of σ_1/σ_N for a lead film with a resistance 197 ohms per square. The bandwidth of each point is ~10%.

mercury films which we have examined. (The transmission through the lead films in the normal state disagreed slightly with the value calculated from the dc resistance. The small ambiguity in the interpretation of the data which this introduces does not affect the qualitative features of the conductivity curve.)

It seems likely that the precursor hump in the conductivity curves for the films is due to the same cause as the dip in the power ratio curve for the bulk samples. For both of the metals under discussion, the structure in the film data and that in the bulk data occur at approximately the same frequency, relative to the frequency at which the main onset of absorption sets in. This structure may be associated with anisotropy of the energy gap. Such anisotropy has been observed in ultrasonic attenuation measurements[4] on tin, and could also account for the observed nonexponential electronic specific heat, as has been pointed out, for instance, by Cooper.[5] The electronic specific heat which is inferred for lead and mercury from critical field data shows a deviation from the exponential which is an order of magnitude greater than that for any of the other superconductors measured.[6,7] This large deviation may well be associated with the marked structure that we observe for these two metals. On the other hand, the structure may be due to the production of collective excitations by the absorption of photons. Anderson has pointed out,[8] on the basis of microscopic considerations, that collective excited states might have energies which lie below the top of the gap predicted by the theory of Bardeen, Cooper, and Schrieffer.[9] The collective excitations may perhaps also be described macroscopically in terms of longitudinal oscillations in the metal. In the case of bulk samples, transverse oscillations may also play a role in explaining the observed effects. Both of these possibilities have been proposed tentatively by Ferrell.[10]

The far-infrared experiments on these and other metals will be reported more fully in forthcoming publications.

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†National Science Foundation Predoctoral Fellow, now at the University of Illinois, Urbana, Illinois.

‡National Science Foundation Predoctoral Fellow, now at the Royal Society Mond Laboratory, Cambridge, England.

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OVERHAUSER EFFECT IN METALLIC LITHIUM

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(Received August 24, 1959)

The enhanced nuclear polarization produced by the saturation of electron spin resonance in metals can be detected in two ways: (a) by observing the enhancement of the nuclear signal, a method widely used since the pioneer work of Carver and Slichter[1]; (b) by observing the shift of the electron spin resonance brought about by the polarization of the nuclei, a method proposed by Overhauser[2] and analyzed by Kaplan[3] but never carried out experimentally so far.

We present a preliminary report of a detection of Overhauser nuclear polarization in lithium metal using both methods. The sample was lithium hydride heavily irradiated by pile neutrons. A sharp (0.3 gauss) electron spin resonance line, observed by Doyle et al.[4] in LiH irradiated with ultraviolet radiation, was attributed by them to conduction electrons in colloidal particles of lithium metal. In our sample, with negligible inhomogeneous broadening, the electron spin resonance line had at room temperature and 10 000 Mc/sec a width of 0.125 gauss (peak to peak of

the absorption derivative). The metallic character of lithium was confirmed by the observation of the nuclear signal of Li^7: the line, with a width of 0.4 gauss, is clearly diffusion-narrowed and presents a relative frequency shift of 0.024%, equal to the known Knight shift in lithium metal. The narrowness and the symmetrical shape of the electron spin resonance line at 300°K, exhibiting the unusual purity and smallness of metallic particles in irradiated LiH, make this sample particularly suitable for Overhauser experiments in high fields and at low temperatures.

(A) Experiments at 300°K: A sample of LiH surrounded by a nuclear magnetic resonance coil was placed inside a microwave cavity fed by a magnetron delivering up to two watts of continuous microwave power. The nuclear magnetic resonance of Li^7 was observed in a field of 3300 gauss while the electron resonance was being partially saturated. Because of the shielding of the sample from the microwave field by the nuclear magnetic resonance coil, no reliable estimate of the field at the sample could be made. The enhancement A of the nuclear polarization is given by

$$A-1=\frac{\gamma_e}{\gamma_{Li^7}}\frac{T_1}{T_{1e}}s=1690\frac{T_1}{T_{1e}}s, \qquad (1)$$

where T_1 is the nuclear spin lattice relaxation time, T_{1e} the relaxation time caused by electron-nucleus coupling only and s the saturation parameter of the electron resonance, related to the microwave power P by $1/s=1+a/P$, where a is a constant depending on the apparatus and the sample. The maximum signal enhancement observed was approximately 100, the uncertainty being caused by the weakness of the unenhanced signal. A plot of $1/A_{obs}$ as a function of $1/P$ was, within experimental error, a straight line from which a value of $A_{max}\simeq 150$ could be extrapolated. This would require $T_1/T_{1e}\simeq 0.1$ rather smaller than the value $T_1/T_{1e}\simeq 0.7$ at room temperature and 3300 gauss deduced from the results of Holcomb and Norberg.[5] This discrepancy may be due to the presence of grains appreciably larger than the electronic (but not the nuclear) skin depth, which would give a full contribution to the natural signal but not to the enhanced signal.

(B) Experiments at 4.2°K: The relative shift D of the electron spin resonance frequency is related to the enhanced nuclear polarization $\langle I_z\rangle$ and to the Knight shift K by

$$D=K\gamma_n\hbar\frac{N}{\chi_p}\langle I_z\rangle\frac{1}{H_0}, \qquad (2)$$

where χ_p is the electron susceptibility, N the number of nuclear spins per unit volume, and $\langle I_z\rangle$ is given by

$$\langle I_z\rangle=\frac{A\gamma_n}{\gamma_e}\frac{\gamma_e\hbar I(I+1)H_0}{3kT}, \qquad (3)$$

where A is the enhancement (1). With $H_0=3300$ gauss, $T=4.2$°K, and $\chi_p=2.1\times10^{-6}$,[6] the absolute shift expressed in gauss is

$$\Delta=DH_0=7.2\,A\gamma_n/\gamma_e. \qquad (4)$$

The electron spin resonance was observed at 10 000 Mc/sec for various microwave levels up to 7 milliwatts, corresponding to an estimated rotating microwave amplitude $H_1\simeq 75$ milligauss, and to a saturation parameter $s\simeq 0.3$ for grains smaller than the skin depth δ. For grains of dimension $d>\delta$, and for such a narrow line, it can be expected, because of electron spin diffusion, that the saturation parameter will be uniform within the grain but reduced by a factor of the order of $(\delta/d)^2$,[7] resulting in a spread in the observed electron spin resonance shifts. As the microwave power is increased, two striking effects occur (Fig. 1): (a) The width and the shape of the line change considerably. (b) The shape depends on the direction of the sweep.

The spread of the curve 1(b), of the order of 2 gauss, agrees with the value of the shift pre-

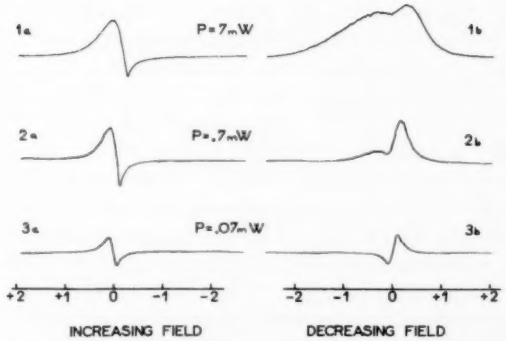

FIG. 1. Recorded electron spin resonance absorption derivative in neutron-irradiated LiH for various power levels P at 4.2°K and 10 000 Mc/sec. One division on the horizontal axis corresponds to one gauss; the sweep is 0.4 gauss/min.

dicted by Eq. (4) for a saturation parameter $s \approx 0.3$, that is for the smallest grains.

The dependence of the line shape on the direction of the sweep is in qualitative agreement with the prediction of Kaplan[3] if the finite nuclear T_1 (10 sec at 4.2°K) and a spread in the size of the grains are taken into account.

The existence of a distribution of electron spin resonance shifts was also exhibited directly: after saturating the electron spin resonance line for several T_1, a large field modulation (5 gauss) was introduced suddenly. The electron line being then saturated during a small fraction of the modulation cycle only, the enhanced nuclear polarization could relax back to approximately its normal value. The electron line observed on the scope presented immediately after the introduction of the modulation, an asymmetrical broadening of the order of 0.4 gauss, which collapsed into a narrow line with a time constant of the order of $T_1 \approx 10$ sec.

The interest of Dr. A. Abragam in this work is gratefully acknowledged.

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ELECTRON COLLISION FREQUENCIES IN NITROGEN AND IN THE LOWER IONOSPHERE*

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(Received July 6, 1959)

Attempts[1,2] to correlate laboratory measurements of electron collision frequencies in nitrogen with the results of measurements using rockets passing through the D-layer of the ionosphere have not been very successful. Our measurements of the electron collision frequencies for thermal electrons in nitrogen, combined with an improved analysis of the data obtained with the rockets, appear to remove the discrepancies.

An improved version of the electron drift velocity tube used by Bradbury and Nielsen[3] has been used to measure the mobility of electrons in nitrogen at such low electric fields that the electrons are in thermal equilibrium with the gas.[4] The measured values of the product of electron mobility, μ, and gas density, N, are $\mu N = 1.10\times10^{24}$, 3.5×10^{23}, and 2.8×10^{23} cm^{-1} volt^{-1} second^{-1} at 77°K, 300°K, and 373°K, respectively. Following Phelps, Fundingsland, and Brown[5] (PFB) these results are analyzed by expressing the reciprocal of the momentum transfer collision frequency, $\nu_m(u)$, as a power series in the electron energy, u, and substituting into the standard expressions for electron mobility[6] to obtain a power series in the most probable electron energy, kT/e. The power series for the mobility is then fitted to the experimental data to obtain the coefficients of the series for $\nu_m(u)$. The resulting $\nu_m(u)$ is shown by the lower solid curve in Fig. 1. The dashed curves show

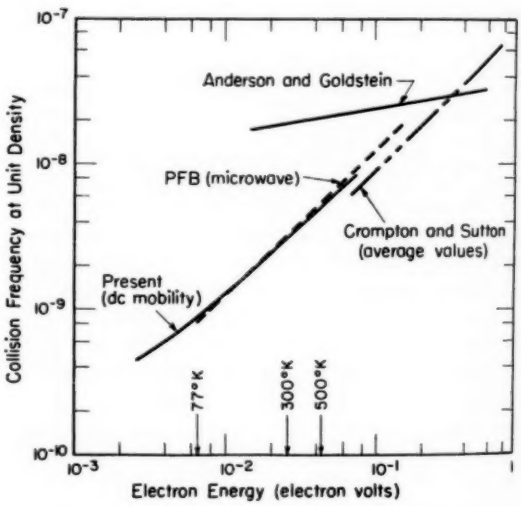

FIG. 1. Momentum transfer collision frequencies for monoenergetic electrons in nitrogen as a function of electron energy. The energy range of the thermal equilibrium experiments is shown by the values of kT/e at which the measurements were made and serves to indicate the range of validity of the curves of collision frequency as a function of energy. The mobility data are found to fit the relation $\mu N = 9.6\times10^{21}(kT/e)^{-1} - 1.39\times10^{19}(kT/e)^{-2}$ cm^{-1} volt^{-1} sec^{-1} within the experimental error so that $N/\nu_m(u) = 8.2\times10^{6}(u)^{-1} - 5.9\times10^{3}(u)^{-2}$ cm^3/sec.

the results of analysis of (a) thermal equilibrium measurements of microwave conductivity by PFB and (b) drift velocity and diffusion measurements at average electron energies above thermal by Crompton and Sutton.[7] The upper solid curve shows the results of Anderson and Goldstein.[8] Figure 1 shows very good agreement between the present results and those of PFB and satisfactory agreement with the data from Crompton and Sutton. These data show that the electron collision frequency in nitrogen is very nearly directly proportional to the electron energy. We shall use this approximation in the following calculations.

In order to evaluate the electron collision frequency in air, we must estimate the collision frequency for electrons in oxygen. The results of analysis of drift velocity and diffusion measurements in oxygen show that the collision frequency for electrons in oxygen is about two-thirds of that for nitrogen for energies down to 0.2 electron volt.[9] We shall assume that the ratio of collision frequencies for oxygen and nitrogen continues to be significantly less than unity down to thermal energies (0.026 ev) so that the error is less than 20% when $\nu_m(u)$ for air is taken equal to that for the nitrogen alone.

The application of the above results to the analysis of the properties of the ionosphere is illustrated by the following re-evaluation of recent data obtained from rockets fired during a polar blackout.[1] First, one notes that because of the energy dependence of the electron collision frequency in nitrogen, relations which assume that the collision frequency is independent of energy should not be used to analyze measurements made in air. In order to facilitate calculations, the function Q used by Kane[1] as a measure of the ratio of the differential absorption to the refractive index is rewritten in terms of functions which properly average $\nu_m(u)$ over the electron velocity distribution.[5,10] Thus,

$$Q(\gamma, Y) = [\sigma_r(\gamma_0) - \sigma_r(\gamma_x)][\sigma_i(\gamma_x) + 1.4\sigma_i(\gamma_0)]^{-1},$$

where $\gamma = \nu_m(u = kT/e)/\omega$, $\gamma_x = \gamma/(1 - Y)$, $\gamma_0 = \gamma/(1 + Y)$, σ_r and σ_i are the real and imaginary parts of the conductivity of the electrons in the gas, $\nu_m(u = kT/e)$ is the collision frequency for monoenergetic electrons at the energy $u = kT/e$, and Y is the ratio of the angular gyrofrequency due to the earth's magnetic field to the angular frequency of the radio wave. Plots of Q vs γ for the value of Y appropriate to Kane's data are shown in Fig. 2 for the cases of collision fre-

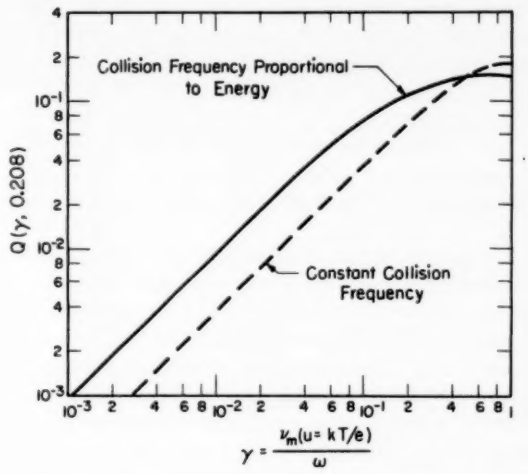

FIG. 2. The function $Q(\gamma, Y)$ vs γ for electron collision frequencies directly proportional to energy and independent of energy. The curves are calculated for $Y = 0.208$ which is the value appropriate to the earth's magnetic field at the site of the rocket measurements and to the frequency (7.75 megacycles/second) used. Values of Q for $\gamma > 1$ are not shown since, for $\gamma^2 \gg 1$, $Q(\gamma, Y)$ becomes extremely sensitive to the exact form of $\nu_m(u)$ at small u.

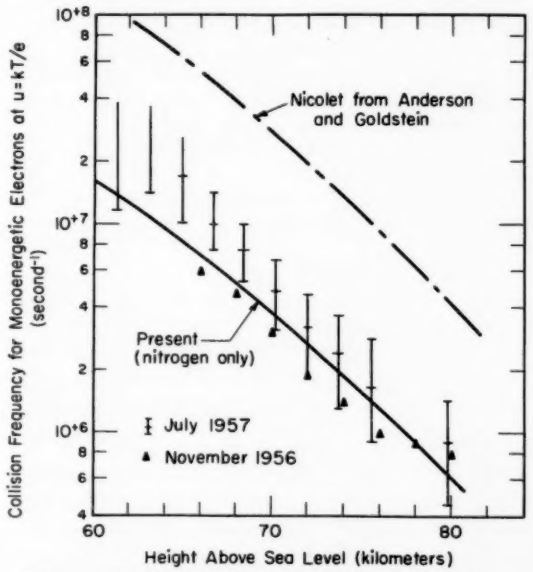
Nicolet from Anderson and Goldstein
Present (nitrogen only)
July 1957
November 1956
Collision Frequency for Monoenergetic Electrons at u = kT/e (second⁻¹)
Height Above Sea Level (kilometers)

FIG. 3. Electron collision frequencies for monoenergetic electrons as a function of height above sea level. The error limits indicated are calculated from those given by Kane and would be approximately the same magnitude for the lower set of points. Note that since the mean values of Q computed from Kane's data for heights of 61.3 and 63.1 kilometers lie above the curve of $Q(\gamma)$ for $\nu_m(u) \approx u$, only the lower limits lead to real values of $\nu_m(u = kT/e)$.

quency directly proportional to energy and independent of energy. The values of $\nu_m(u=kT/e)$ inferred from a given observed value of Q for the two different energy dependences of $\nu_m(u)$ are in the ratio of 2.5:1 at very small values of ν_m/ω. Note, however, that since this ratio changes with ν_m/ω one cannot use the Q curve for constant ν_m shifted to the left by a factor of 2.5.

The points of Fig. 3 show the collision frequencies for monoenergetic electrons in the D-layer computed from Fig. 2 and from Kane's values of Q. The curves give the collision frequencies for monoenergetic electrons in nitrogen at the gas densities and temperatures given by Nicolet[2] using our $\nu_m(u)$ (lower curve) and using Nicolet's extrapolation of results of Anderson and Goldstein[2] (upper curve). The agreement between the lower curve and the points is within the scatter of the rocket data. This agreement can be considered as evidence for the correctness of the gas density and temperature data obtained from other rocket studies.[2]

*This work was supported in part by the Advanced Research Projects Agency, the Office of Naval Research, and the Air Force Special Weapons Center.

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MOMENTS OF INERTIA OF EVEN-EVEN RARE EARTHS*

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Belyaev[1] has considered the effects of nuclear pairing interactions in analogy with the theory of superconductivity.[2-4] His treatment leads to the following formula[1] for the moment of inertia of an even-even nucleus:

$$\mathfrak{I}=\hbar^2\sum_{kl}\frac{|\langle k|J_x|l\rangle|^2}{E_k+E_l}\left[1-\frac{\Delta^2+(\tilde{\epsilon}_k-\lambda)(\tilde{\epsilon}_l-\lambda)}{E_kE_l}\right], \quad (1)$$

where

$$E_k=[(\tilde{\epsilon}_k-\lambda)^2+\Delta^2]^{1/2}, \quad (2)$$

and the sums, l and k, run over all single particle states of a self-consistent spheroidal well. The "chemical potential" λ is the solution of the equation

$$\sum_k\{1-[(\tilde{\epsilon}_k-\lambda)^2+\Delta^2]^{-1/2}[\tilde{\epsilon}_k-\lambda]\}=N. \quad (3)$$

N here is the number of neutrons or protons outside of closed shells. The quantity Δ is equal to one-half the induced energy gap.

Numerical computations based on Eq. (1) have been made for twenty-six rare earth nuclei which

Table I. Rotational energies of even-even nuclei. Columns one to three list the nuclei considered, their deformations,[a] and their assigned energy gap parameters[b] which yield the calculated energy of the first rotational (2+) state in kev (Column 5). Columns 4 and 6 give the observed rotational energy[a,c] and the percent difference between experiment and theory. The energy gap is given in units of $\hbar\omega_0{}^0$, which is approximately $41A^{-1/3}$ Mev for a nucleus of mass number A.

Nucleus	δ_{Coulomb}	$\Delta/\hbar\omega_0{}^0$	E_{exp} (kev)	E_{theor} (kev)	% error
$_{60}Nd_{90}{}^{150}$	0.24	0.139	130	139.5	+ 6.8
$_{62}Sm_{90}{}^{152}$	0.27	0.139	122	123.0	+ 0.8
$Sm_{92}{}^{154}$	0.31	0.128	83	99.7	+16.7
$_{64}Gd_{90}{}^{154}$	0.28	0.139	123	120.0	- 2.5
$Gd_{92}{}^{156}$	0.39	0.128	89	85.6	- 4.0
$Gd_{94}{}^{158}$	0.44	0.117	79	68.3	-15.7
$Gd_{96}{}^{160}$	0.45	0.108	76	66.1	-15.0
$_{66}Dy_{94}{}^{160}$	0.33	0.117	86	92.1	+ 6.6
$Dy_{96}{}^{162}$	0.34	0.108	82	84.4	+ 2.8
$Dy_{98}{}^{164}$	0.39	0.100	73	70.0	- 4.3
$_{68}Er_{96}{}^{164}$	0.31	0.108	90	88.3	- 1.9
$Er_{98}{}^{166}$	0.31[d]	0.100	80	85.9	+ 6.9
$Er_{100}{}^{168}$		0.093	80	83.7	+ 4.4
$Er_{102}{}^{170}$		0.088	79	77.7	- 1.7
$_{70}Yb_{100}{}^{170}$	0.28	0.093	84	86.1	+ 2.4
$Yb_{102}{}^{172}$	0.29[d]	0.088	78	80.1	+ 2.6
$Yb_{104}{}^{174}$		0.085	76	77.1	+ 1.4
$Yb_{106}{}^{176}$		0.085	82	74.7	- 9.8
$_{72}Hf_{104}{}^{176}$	0.28	0.085	89	83.8	- 6.2
$Hf_{106}{}^{178}$	0.29	0.085	91	78.5	-15.9
$Hf_{108}{}^{180}$	0.26	0.086	93	90.3	+ 3.0
$_{74}W_{108}{}^{182}$	0.25	0.086	100	101.9	+ 1.9
$W_{110}{}^{184}$	0.23	0.090	112	110.9	- 1.0
$W_{112}{}^{186}$	0.23	0.096	124	118.3	- 4.8
$_{76}Os_{110}{}^{186}$	0.19	0.090	137	130.9	- 4.8
$_{76}Os_{112}{}^{188}$	0.17	0.096	155	166.8	+ 7.1

[a]See reference 8.
[b]See reference 7.
[c]See reference 9.
[d]These deformations were obtained from measurements on unseparated isotopes.

exhibit rotational spectra. The eigenvalues, $\tilde{\epsilon}_\nu$, and wave functions of the self-consistent field were taken from the work of Nilsson,[5] except for certain modifications indicated by a recent study of the spectra of odd-mass nuclides.[6] The energy gaps were obtained from a smooth curve drawn through the experimental pairing energies of Johnson and Bhanot,[7] and the quadrupole deformations, from Coulomb excitation studies.[8]

The results are summarized in Table I. They indicate that Eq. (1), together with the Nilsson model, provides a remarkably good description of the moments of inertia in the rare earths, thereby lending support to the superconductor theory of nuclear pairing effects.

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NEW MODEL FOR VIBRATIONAL SPECTRA IN EVEN-EVEN NUCLEI*†

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The vibrating surface model,[1] with its prediction of equal level spacings and the spin sequence[2] 0, 2, and a triad 0, 2, and 4, is the simplest among those presented in explaining the vibrational spectra of even-even nuclei. Experimentally observed spectra, however, indicate deviations from it among which the more important ones are[3]: (*i*) The degeneracy of the triad is lifted and usually[4] $E_2(2) < E_1(4)$; (*ii*) the ratio $E_2(2)/E_1(2)$ is generally greater than two.

The first approach to explain these deviations was proposed by Scharff-Goldhaber and Weneser[3] who considered the coupling of single particles to the simple surface vibrational motions. This model gives rule (*ii*) above, but for (*i*) it predicts $E_2(2) > E_1(4)$. Recently Raz[5] found, by extending the GW calculation, that it was possible to satisfy also (*i*); however, as will be discussed below, this treatment appears to possess other difficulties.

Wilets and Jean[6] proposed a model very similar to the simple surface vibrational model, but which differs in that a finite, shape-independent equilibrium deformation is assumed; (the γ-unstable model). It explains (*ii*) but gives $E_2(2) = E_1(4)$.

Figure 1(a) shows the Cd^{114} spectrum obtained experimentally by Motz[7] in which all members of the triad as well as the 3_1 state were observed; (this latter may be considered as a member of the third excited surface vibrational state). This spectrum bears out (*i*) and (*ii*) and in addition indicates that (*iii*) $E_2(0) \approx E_1(4) \approx E_2(2)$, and (*iv*) the ratio of $E_1(3)/E_1(2)$ is close to, but somewhat larger than three. The calculation of Raz predicts too large a value for $E_2(0)$ and for $E_1(3)$ and thus fails to reproduce the new rules (*iii*) and (*iv*).

The WJ model is consistent with (*ii*) and (*iv*), but is inconsistent with (*i*) and (*iii*). The inconsistency, however, is not too drastic and this fact suggests that with a slight modification of the model these discrepancies might well be removed. By examining the explicit form of the wave functions of each state in the WJ model, it is not hard to see that the desired change in the spectrum will be obtained by introducing a potential energy term of the form[8]

$$\sum_n k_n (\beta - \beta_n)^n \cos^n 3\gamma, \qquad (1)$$

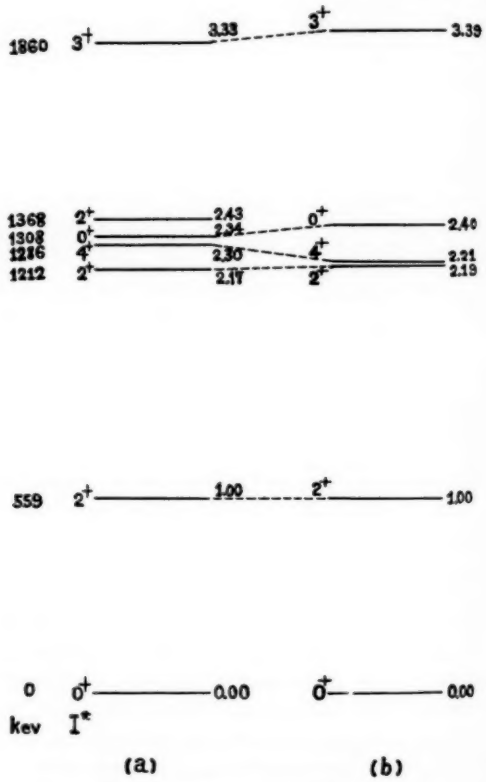

FIG. 1. Spectrum of the lower states of Cd^{114}. (a) Experimental; (b) theoretical. On the left-hand side of (a) are given the excitation energy in kev and the spin of each state, while on the right-hand side the ratio $E_n(I)/E_1(2)$ is given. The 2+ state at 1368 kev seems to be due to a single-particle excitation and the absence of the corresponding state in (b) is without significance since we here consider only collective states.

to the WJ potential

$$\tfrac{1}{2}C(\beta-\beta_0)^2. \qquad (2)$$

The spectrum obtained with the appropriate choice of the parameters in (1) and (2) is shown in Fig. 1(b) and it is seen that, in fact, good agreement with experiment is found.[9] The relative intensities of electromagnetic transitions between several states are also calculated and compared with experiment[10] in Table I, and again good agreement is obtained.

We now discuss the justification for introducing the term (1). For this purpose we first note that the WJ potential (2) can be described by curve (a) of Fig. 2, where the right and left halves, respectively, correspond to $\gamma=0°$ and $\gamma=60°$ (i.e., prolate and oblate deformations). Since the potential (2) is γ-independent, curve (a) has a right-left symmetry. A potential energy curve which is consistent with the parametric values (summarized in Table II) used in the present calculation is shown as curve (b) in Fig. 2. This curve has a right-left asymmetry which is clearly the result of the addition of the term (1) to the WJ potential. If we were to derive this type of potential from "first" principles, we should have taken, e.g., the Nilsson scheme[11] and added the single-particle energies for various values of the nuclear deformations. With such a procedure

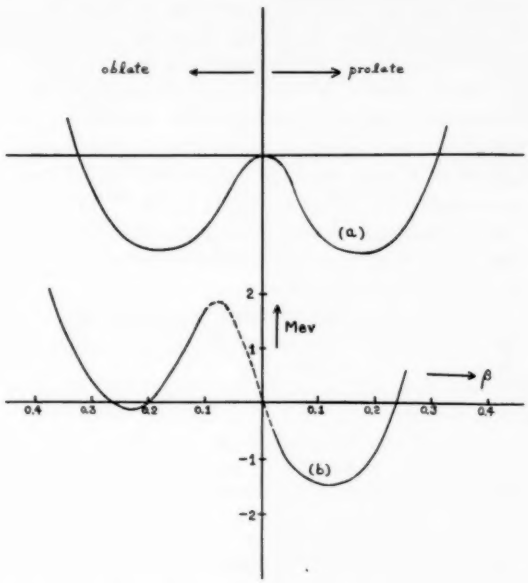

FIG. 2. Potential energy curve in various approximations. (a) Schematic representation of the WJ potential (2). (b) Potential energy curve which is consistent with the parametric values given in Table II, and which produces the theoretical spectrum (b) of Fig. 1.

Table I. Comparison of the experimental and the theoretical values of the relative reduced transition probabilities of γ rays between several states. All the γ rays involved are $E2$ and therefore are not specified.

	Experimental	Theoretical
$T(2_2\rightarrow 2_1)/T(2_1\rightarrow 0_1)$	1.37 ± 0.31	1.70
$T(2_2\rightarrow 0_1)/T(2_1\rightarrow 0_1)$	0.020 ± 0.003	0.014

Table II. The parametric values used to obtain the theoretical spectrum of Fig. 1(b). In determining these values use has been made of the half-life of the 2_1 state (see reference 10) as well as the energy $E_1(2)$ shown in Fig. 1(a). For the $n=2$ term of (1), it is found that a very small value for k_2 yields good agreement with experiment; therefore, we show here the extreme case of $k_2=0$.

C	k_1	β_0	β_1
197 Mev	11.6 Mev	0.176	0.235

an asymmetric energy curve is expected and thus would seem more natural than a symmetric one. To get a curve which can be compared directly with curve (b) of Fig. 2, however, it is necessary to take into account the residual interaction and such a calculation is now under way.

Regarding vibrational nuclei other than Cd^{114}, we first note that the ratio $E_2(2)/E_1(2)$ is less than two in several Hg and Pt isotopes. The recent theory of Davydov and Filippov[12] predicts values for this ratio that always exceed two, but in our model ratios less than two are easily obtained by allowing negative values for β_0 in (2). The DF theory further predicts that $E_1(2)+E_2(2)=E_1(3)$, while in our theory $E_1(2)+E_2(2)$ is greater than or less than $E_1(3)$ for $E_2(2)/E_1(2)$ greater than or less than 2, respectively. To the extent that only for Cd^{114} and Pt^{192} are data available [in the region where $E_2(2)/E_1(2)$ does not deviate very much from two], we find that this latter prediction agrees with experiment.

Finally, it should be mentioned that the DF theory can say nothing about the 0_2 state, and while $E_2(2)$ is correctly given, it predicts a value for $E_1(4)$ in Cd^{114} which is somewhat too

large. DF stress the agreement of their theory with experiment regarding the relative intensities of the $M1$ and $E2$ transitions between the 2_1 and 2_2 states, in addition to the relative intensities of other $E2$ transitions. However, it might be much more reasonable to consider the effect of single-particle excitations which begin to assume importance at this point.[13] This matter, however, will be discussed elsewhere together with a more detailed account of the context of the present note.

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‡Now at the Department of Physics, Tokyo University of Education, Tokyo, Japan.

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[2]In this note we consider exclusively states which have even parity; therefore, we will not specify the parity of any state here and in the following.

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[4]Throughout this note $E_n(I)$ is the energy, above the ground state, of a state I_n, n labelling the position, energy-wise, of the particular state in question among all the others with the spin I.

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[6]L. Wilets and M. Jean, Phys. Rev. 102, 788 (1956); referred to as WJ.

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[8]For the notation β and γ see, e.g., reference 6. k_n, β_n, and C are constant parameters whose magnitudes are adjusted to produce agreement of the spectrum with experiment.

[9]In this calculation we considered only the first two terms of (1); i.e., we set $k_n = 0$ for $n \geq 3$. This is not, however, too bad an approximation because as is easily seen all the n = odd (and/or even) terms act, in so far as the lower states are concerned, in a similar way to produce the desired modification of the WJ spectrum. The two terms in (1) remaining are treated as the perturbations to the WJ Hamiltonian with the states $N \leq 4$ treated explicitly, and those with $N \geq 5$ accounted for by a closure argument. (The definition of N is the same as in reference 6.)

[10]P. H. Stelson and T. K. McGowan, Phys. Rev. 110, 489 (1958); Bull. Am. Phys. Soc. 2, 267 (1957).

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[13]In the derivation of Eq. (3.2) of DF, they expressed the nuclear shape by taking up to the quadratic term in α to obtain a nonvanishing $M1$ transition probability. This might not be consistent with the assumption that the deformation is described by an ellipsoid. If the latter had included also the 2^4-th pole deformation, DF would have obtained a vanishing $M1$ transition probability. We are indebted to Professor B. R. Mottelson for a discussion on this point.

ANGULAR DISTRIBUTIONS FROM STRIPPING REACTIONS OF LOW Q VALUES*

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Proton groups from the reactions $Li^7(d,p)Li^8$ and $C^{12}(d,p)C^{13*}$ (3.09-Mev state) for a range of deuteron energies between 0.5 and 2.5 Mev have been studied using high-resolution magnetic analysis. In fitting the measured angular distributions for the protons to a simple stripping theory, uncorrected for Coulomb and other perturbing effects, an unusually high degree of agreement was found.

Wilkinson[1] suggested that such agreement might be expected for reactions of fairly low Q values.

It is well known that the deviations of measured stripping (deuteron-induced) reactions from the simple Butler[2] and Born-approximation[3] theories for moderate energies (≥ 10 Mev, say) take a characteristic form. In general the first maximum can be unambiguously matched, but with increasing angle the agreement soon deteriorates. In particular the well-defined first minimum in the theoretical curves is filled in and subsequent maxima are poorly defined. In fact some 20 or so degrees after the first maximum the measured intensity is normally persistently higher than the theoretical prediction. This is attributed to competing compound-nucleus formation. The perturbing effects of nuclear and Coulomb inter-

actions also play their role in distorting the simple comparison.

At low deuteron energies the deviation of measured distributions from simple theoretical ones becomes more serious. This is not unexpected as the effect of compound-nucleus formation and interaction effects are anticipated to be more prominent.

In order to improve agreement between experiment and theory, many modifications have been made to the simpler formalisms. Among these can be mentioned the work of Friedman and Tobocman,[4] Daitch and French,[5] Horowitz and Messiah,[6] Grant,[7] Tobocman,[8] and Tobocman and Kalos.[9] These refinements show that the effect of considering the Coulomb interaction is to displace the angular distributions toward larger angles, to broaden the peaks and fill in the valleys, and to reduce the total cross section. On the other hand, the effect of introducing nuclear interactions is to displace peaks towards smaller angles, to sharpen these peaks, and to reduce the total cross section. The distortion of simple stripping curves which can be achieved is strikingly great. It is unfortunate that when allowing for all possible effects, agreement with experiment is readily obtained, so that it is impossible to say whether the parameters employed to achieve such agreement have any particular physical significance. The conclusion is drawn that in order to avoid having to make hypotheses on the values of these parameters, experiments should be performed at energies well above the Coulomb barrier.

Two examples of the results obtained for the $Li^7(d,p)Li^8$ (ground state) reaction with Q value of -0.188 Mev are given in Fig. 1. These were obtained for incident deuteron energies of 1.9 and 1.5 Mev, respectively.

Another reaction of small Q value has been studied, viz., $C^{12}(d,p)C^{13*}$ (3.09-Mev state). The Q value is -0.367 Mev. An example of the results obtained in an angular distribution, for an incident deuteron energy of 2.1 Mev, is given in Fig. 2.

The theoretical curves associated with the measurements given in these diagrams were obtained from the non-Coulomb, noninteraction stripping theory formalism of Friedman and Tobocman,[4] using for convenience the tables and graphs of Enge and Graue,[10] based on this formalism. Other stripping derivations have also been tried, also with striking agreement.

Excitation functions have been measured for

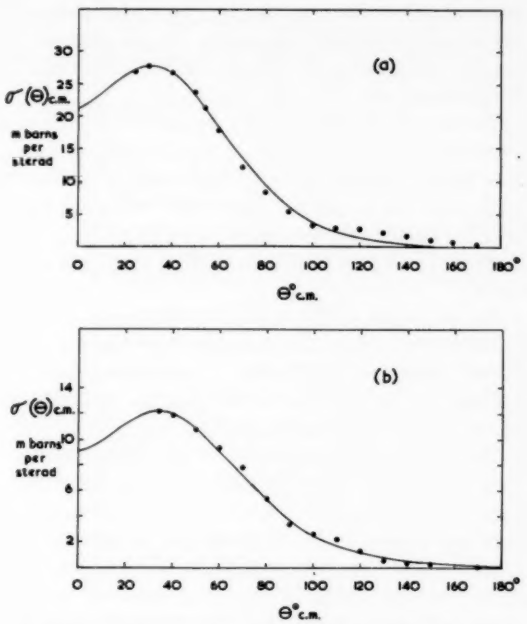

FIG. 1. Angular distribution of protons from the reaction $Li^7(d,p)Li^8$. (a) $E_d = 1.9$ Mev. The points represent experimental observations; the solid curve represents a calculated stripping distribution with $R = 6.0 \times 10^{-13}$ cm and $l = 1$. (b) $E_d = 1.5$ Mev. The points represent experimental observations; the solid curve represents a calculated stripping distribution with $R = 6.5 \times 10^{-13}$ cm, and $l = 1$.

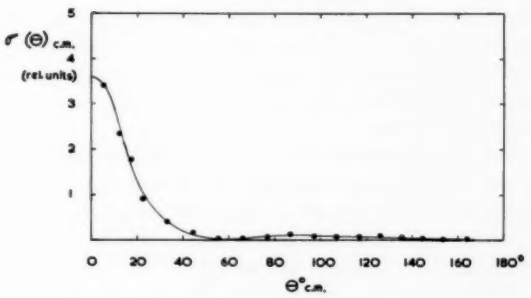

FIG. 2. Angular distribution of protons from the reaction $C^{12}(d,p)C^{13*}$ (3.09-Mev state) $E_d = 2.1$ Mev. The points represent experimental observations: the solid curve represents a calculated stripping distribution with $R = 7.4 \times 10^{-13}$ cm, and $l = 0$.

both reactions, $Li^7(d,p)Li^8$ (ground state) and $C^{12}(d,p)C^{13*}$ (3.09 Mev). In the lithium case one resonance was closely studied. Angular distributions were measured at the resonance energy, and at energies closely flanking the resonance.

No change—other than the increase in intensity associated with the resonance—in the marked degree of agreement between experimental and theoretical distributions was found.

In the carbon experiment a number of resonances were found and the effect of a particularly strong one in the angular distributions was studied in the same way as in the lithium case. Here too there was no significant change in the degree of goodness of fit with the simple stripping formalism used.

On the basis of these two experiments it would indeed appear that for small-Q reactions at low bombarding energies there is particularly little influence on the shape of the angular distributions from Coulomb and nuclear effects.

The physical picture underlying this proposal is as follows.[11] The momentum of the outgoing proton in a (d,p) reaction is made up of about half the momentum of the ingoing deuteron and of a part of the internal momentum of the deuteron at the instant at which stripping occurs. In the case of a large-Q reaction the total proton momentum must be high. If, however, the proton's share of the momentum of the ingoing deuteron is small—when the deuteron energy is small—the large deficit in momentum must be obtained from the ground-state wave function of the deuteron. In order to get this large latter contribution, the separation of proton and neutron in the deuteron must be small at the instant of stripping. For a small-Q reaction at low energy, on the other hand, the contribution to the outgoing proton momentum from the ground-state deuteron wave function need not be large, so that the separation of proton and neutron can be far greater than in the high-Q case.

The clear implication of this is that for small-Q stripping at low energies the proton can be well removed from the target nucleus when stripping occurs, reducing radically thereby the perturbing nuclear and Coulomb effects.

Wilkinson[11] has pointed out that in such small-Q, low-energy reactions, competition through the formation of a compound nucleus will in fact be discouraged, since the proton which—as has been indicated—can be well removed from the target nucleus for stripping, will have to penetrate the Coulomb barrier if a compound nucleus is to be formed. As in the high-Q case the proton must at least partially penetrate the Coulomb barrier in order to get sufficiently near to the neutron, this effect is much reduced.

The examples presented in this paper indicate an unusually high degree of agreement between experimental angular distributions and a simple stripping theory uncorrected for Coulomb and other effects. A physical description can give a clear connection between this agreement and the fact that such reactions have low Q values and are performed at low bombarding energies.

A full description and analysis of both experiments and their results will be published in the near future.

The author wishes to express his gratitude to Mr. E. S. Shire for suggesting a study of the competition between the stripping and compound-nucleus mechanisms for the reaction $Li^7(d,p)Li^8$, to Professor D. H. Wilkinson for considerable guidance and encouragement, and Dr. R. Eden for many stimulating discussions. He would also like to place on record his appreciation for financial support from the Shell Petroleum Company, and from the South African Council for Scientific and Industrial Research.

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*This work was done in the Cavendish Laboratory, University of Cambridge, Cambridge, England.

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MEASUREMENT OF μ^+ LIFETIME

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(Received September 10, 1959)

The conservation of the weak-coupling vector current in the presence of strong interactions[1] is a hypothesis which still lacks convincing positive experimental verification. As a consequence of this hypothesis a lepton-pion coupling arises which is large when the available energy is large as in muon capture or short-lived β decay. In μ^- capture the effect amounts to 20-30% of the rate, but it seems unfortunately to be quenched by an equally large pseudoscalar coupling induced by virtual π-meson emission.[2] Recent experiments on muon capture between the ground states of C^{12} and B^{12} support, but do not prove, these conclusions.[3] In β decay, relativistic effects and radiative corrections tend to eclipse the effect of the mentioned lepton-meson coupling.

The remarkable agreement of the experimental with the "theoretical" value for the lifetime of the muon (as stressed by Feynman and Gell-Mann[1]) is therefore the strongest indication that the vector current is conserved and the strengths of axial and polar vector currents are equal when strong interactions are not present. In view of this, we decided, in the spirit of experiments involving weak interactions, to attempt to bring the experimental value of the μ^+ lifetime (2.22 ± 0.02 μsec)[4] even closer to the "theoretical" value (2.31 ± 0.05 μsec).[5] Meanwhile, Swanson et al.[6] have reported a measurement where they succeeded in this, their new value being $\tau(\mu^+) = 2.261 \pm 0.007$ μsec.

The philosophy of our experiment was to elaborate on the detection equipment for muons and their decay electrons, and to read their time difference in a simple manner from pulses on an oscilloscope film. We used 30 hours running time of the CERN cyclotron to set up the experiment and expose the films, while it took us about 300 hours to read and analyze 10^5 traces.

The experimental arrangement is shown in Fig. 1. A 180-Mev/c meson beam is momentum-analyzed by two successive bending magnets after leaving the fringing magnetic field of the cyclotron. The velocity-selecting Čerenkov counter 1 in coincidence with the scintillation counters 2 and 3 registered the passage of a μ^+. Counter 1 consisted of a 10-cm thick cylinder subdivided along the beam direction into 5 cells which were filled with different mixtures of water and ethylene glycol.[7] The decrease in the velocity of the particles by slowing down is thus compensated for by an increase in the index of refraction (from 1.33 to 1.37) so as to keep the angle of the Čerenkov light refracted out of the radiator constant. A ring image, corrected for spherical, astigmatic, and some of the chromatic aberrations, was formed on four 56 AVP photomultipliers by reflecting the light in a nearly spherical mirror and passing it through a ring-shaped Lucite lens. The detection efficiency of the Čerenkov counter for muons was 98%, while for π mesons and electrons of the same momentum it was less than 10^{-5}.

The muons were on the average stopped in the middle of counter 4, which was a Plexiglas cylinder 5 cm thick and 10 cm in diameter. But for the side facing the photomultiplier (RCA C7071) the radiator had a reflective coating of evaporated aluminium, thus making it 4π-sensitive. The counter was surrounded by two layers of 1-mm thick mu-metal, which reduced the

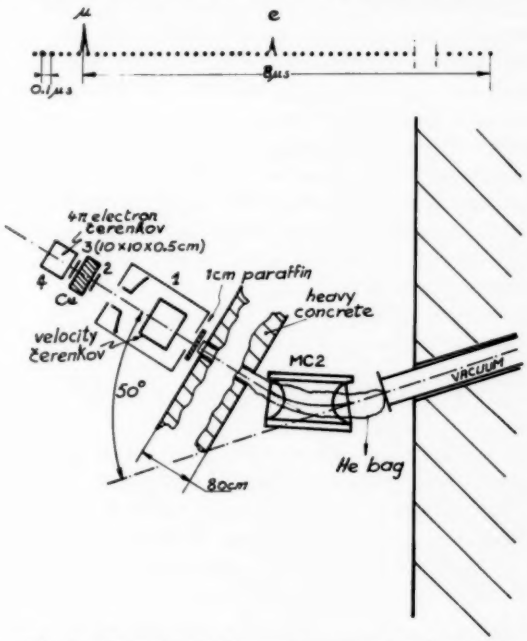

FIG. 1. Experimental arrangement and a facsimile of a typical oscilloscope trace.

magnetic field in the interior to of the order of 10^{-2} gauss.

A coincidence between counters 1, 2, and 3 with 4 in anticoincidence (1-2-3-4) started the sweep on a 445 Tektronix oscilloscope. The intensity of the oscilloscope beam was modulated with a 10-Mc temperature-controlled crystal oscillator. Thus a picture as shown in Fig. 1 appears on the screen of the oscillograph. The pulse from the electron counter 4 in anticoincidence with 2 (4-$\bar{2}$) is shown as a later pulse. The accidental counting rate is obtained from the frequency of pulses appearing at times earlier than the μ pulse, and from the "doubles" rate. The latter consists of pulses from a muon, its decay electron, and an accidental. We had a total of 581 such doubles, and by analyzing their time variation along the sweep the corresponding variation in the accidental coincidence rate could be determined. We considered such an analysis necessary since the intensity pattern of the beam pulses from a cyclotron can be very asymmetrical, but we found a flat distribution over the measured time interval.

Every third trace on the film had an electron pulse. The accidental rate was 0.8% of the counting rate in the first channel. The film was scanned with the help of a microfilm reader which made it possible to locate the position of the pulses to about 0.02 μsec. The scanning was performed and controlled by five physicists and four technicians, and we have not been able to discover any systematic errors. The least-squares treatment of the data was done with the CERN Mercury computer. The results of our measurements are shown in Fig. 2. Our value for the mean life (2.20 ± 0.015 μsec) disagrees with the recent result of Swanson et al.,[6] and agrees with the classical value of Bell and Hincks.[4] The error has arbitrarily been increased from the statistical error ($\frac{1}{2}$%). Previous experiences with similar measurements of lifetimes seem to indicate that errors of less than 1% require the convergence of results of different experimental methods.

We gratefully acknowledge the constant experimental help of P. Duteil and R. Thill and the assistance in film reading of P. Meuret and M. Namias.

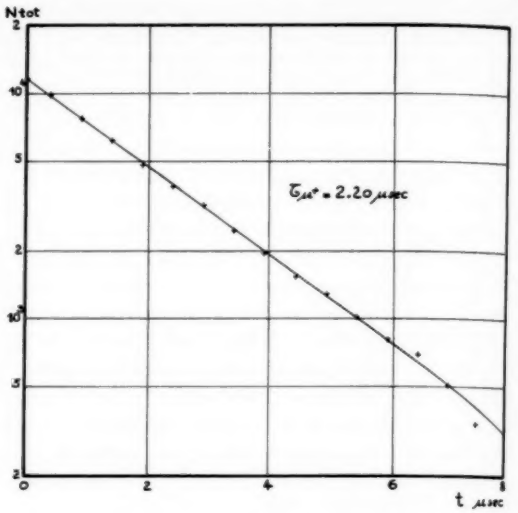

FIG. 2. Decay curve of the muon (background subtracted).

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GAUGE DEPENDENCE OF THE WAVE-FUNCTION RENORMALIZATION CONSTANT IN QUANTUM ELECTRODYNAMICS*

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We should like to point out the existence of an exact and simple relation between the electron Green's function renormalization constants in the general class of "manifestly" covariant gauges. We consider those gauges where the unrenormalized zeroth-order photon Green's function in momentum space has the form

$$D_{\mu\nu} = D_{\mu\nu}{}^{0} - k_\mu k_\nu \lambda(k), \tag{1}$$

where λ is an arbitrary function of k. By $D_{\mu\nu}{}^{0}$ we shall mean a covariant Green's function in a fixed gauge.

One can show that the exact, unrenormalized electron Green's functions in the respective gauges are connected by the relation

$$G_\lambda(x-x') = \exp\{ie_0{}^2[\lambda(x-x') - \lambda(0)]\}G_0(x-x'), \tag{2}$$

where e_0 is the unrenormalized charge of the electron, and

$$\lambda(x) = \int \frac{(dk)}{(2\pi)^4} e^{ik\cdot x}\lambda(k). \tag{3}$$

This result has been derived previously,[1] but in a manner which seems to us somewhat questionable. An independent and rigorous derivation is possible using the techniques of external sources, and is given in papers on the gauge group in electrodynamics.[2,3] The authors of these latter papers were motivated by somewhat different considerations. It should also be marked that the connection between the Green's function in the radiation gauge [which does not fall into class (1)] and the Lorentz gauge has been derived[4] by an alternative method which should also apply to the case under consideration.

If the transformation is nonsingular, the function $\lambda(x)$ vanishes as $x^2 \to \infty$. Further, if the singularity of G at $p^2 = -m^2$ is made rigorously a pole by cutting off the soft photons with an invariant photon mass μ, then asymptotically in any gauge

$$G(x-x') \to Z_2 G_m(x-x') \tag{4}$$

as $(x-x')^2 \to \infty$, where

$$G_m(x-x') = \int \frac{(dp)}{(2\pi)^4} \frac{e^{ip\cdot(x-x')}}{\gamma\cdot p + m}, \tag{5}$$

since m, the physical mass of the electron, is the lowest invariant frequency contained in $G(x-x')$. Z_2 is the so-called wave-frequency renormalization constant. It follows from Eq. (2) that the wave-function renormalization constants in the two gauges are connected by the relation

$$Z_2{}^{\gamma} = \exp\{-ie_0{}^2\lambda(0)\}Z_2{}^{0}. \tag{6}$$

To give an example of this relation, one may take for $D_{\mu\nu}{}^{0}$ the function

$$D_{\mu\nu}{}^{0} = \delta_{\mu\nu}\left(\frac{1}{k^2+\mu^2} - \frac{1}{k^2+\Lambda^2}\right), \tag{7}$$

where μ is the photon mass and Λ is inserted so that we may regard local electrodynamics as the limit of a cutoff theory. Then we choose for λ the function

$$\lambda = \gamma\left(\frac{1}{k^2+\mu^2} - \frac{1}{k^2+\Lambda^2}\right)\frac{1}{k^2}, \tag{8}$$

so that we study the one-parameter family of gauges for which

$$D_{\mu\nu} = \left(\delta_{\mu\nu} - \gamma\frac{k_\mu k_\nu}{k^2}\right)\left(\frac{1}{k^2+\mu^2} - \frac{1}{k^2+\Lambda^2}\right) \tag{9}$$

In this case, $\lambda(0) = i(\gamma/8\pi^2)\ln(\Lambda/\mu)$; therefore

$$Z_2{}^{\gamma} = (\Lambda/\mu)^{\gamma\alpha_0/2\pi} Z_2{}^{0}, \tag{10}$$

where $\alpha_0 = e_0{}^2/4\pi$ is the unrenormalized coupling constant.

If Λ is allowed to approach infinity, it is clear that $Z_2{}^{\gamma}$ can approach a finite and nonvanishing limit at most in one of the gauges of class (9). Thus, if as $\Lambda \to \infty$,

$$Z_2{}^{0} \to (\Lambda/\mu)^{-\epsilon\alpha_0/2\pi} Z_2{}^{f},$$

where Z_2^f is finite and not zero, then in the gauge in which $\gamma = \epsilon$ the renormalization constant would be finite and equal to Z_2^f in this limit. However, we wish to stress that even if such a special gauge did not exist, these considerations would in no way affect the possibility that the unrenormalized physical theory is consistent. Indeed, the reason that a simple relation of the type (2) exists is that the gauge change refers to the way in which the longitudinal and scalar quanta are coupled to the electron, and these quanta (because of the conservation of the fluctuating vacuum currents) enter in an essentially noninteracting way. The factor $(\Lambda/\mu)^{\gamma\alpha_0/2\pi}$ is produced by the change in the number of these quanta in the state in which $p^2 = -m^2$, and consequently has nothing to do with the physical theory, which is concerned only with the transverse quanta. Therefore, the fact that these degrees of freedom introduce inconsistencies only indicates that the use of formulations of electrodynamics that employ them is dangerous in the investigation of the consistency of the (unrenormalized) physical theory at high energies. This points to the radiation gauge as useful for these purposes since it does not fall into class (1), unphysical quanta are avoided, and the Green's functions are directly related to matrix elements of operators in a Hilbert space with a physically acceptable (positive) metric.

In spite of these difficulties, which have to do with the consistency of the local theory, one may still make use of the gauges of class (9) in a pragmatic way provided that Λ is chosen larger than any energy of physical interest. In this case it is useful to note that information about the infrared structure of the Green's function in various gauges may be derived from Eq. (2). It should be stressed again that changing the gauge alters only the number of scalar and longitudinal quanta in the various states and hence has nothing to do with physics. However, simplifications in calculations may occur if the gauge is chosen properly. In the low-energy domain (as opposed to the infinite-energy domain) the transverse quanta should enter in an essentially noninteracting way once the charge is renormalized. It is therefore not inconceivable that cancellations between real quanta and the longitudinal and scalar quanta could be arranged so that a gauge of type (9) might exist where G has rigorously a pole at $p^2 = -m^2$, in the limit as $\mu \to 0$. In this case, the singularity in any other gauge can be calculated quite simply by using (2).

If we allow $\mu \to 0$, the difference $ie_0^2[\lambda(x) - \lambda(0)]$ remains finite and becomes equal to

$$-\frac{i\gamma\alpha_0}{2}\left\{\frac{H_1^{(2)}(\Lambda(-x^2)^{1/2})}{\Lambda(-x^2)^{1/2}} + \frac{i}{\pi}\ln[\tfrac{1}{2}\gamma\Lambda(-x^2)^{1/2}] - \frac{i}{2\pi} - \frac{1}{2}\right\}.$$

Therefore, for $\Lambda(-x^2)^{1/2} \gg 1$,

$$ie_0^2[\lambda(x) - \lambda(0)] = (\gamma\alpha_0/2\pi)\ln[m(-x^2)^{1/2}] + \text{const.} \quad (11)$$

Suppose now that there exists a gauge with $\gamma = \bar{\gamma}$, where the Green's function has a pole at $p^2 = -m^2$. Then it is possible to show that in the gauge corresponding to an arbitrary value of γ, the Green's function in the neighborhood of $p^2 = -m^2$ will behave like

$$\frac{1}{\gamma\cdot p + m}\left\{\frac{m^2}{p^2 + m^2}\right\}^{\alpha_0(\gamma - \bar{\gamma})/2\pi} \quad (12)$$

The work of Yennie[5] indicates that such a gauge exists with

$$\bar{\gamma} = 1 - 3(\alpha/\alpha_0), \quad (13)$$

where α is the renormalized coupling constant. This is explicitly verified to the fourth order by Solov'ev.[6]

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ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

PAIR DISTRIBUTION FUNCTION AND TWO-BODY PROPAGATOR. S. Fujita, Department of Physics, Northwestern University, Evanston, Illinois (Received April 27, 1959).

Starting from the ordinary definition of the pair distribution function, the following theorem is proved without the help of the cluster integral expansion and "toron" diagrams: The pair distribution function in the grand canonical ensemble can be expressed in terms of the two-body propagator corresponding to the scattering in position-reciprocal temperature space. A simple expression for the internal energy of a nonideal gas with pair interactions is obtained in terms of the pair distribution function.

STATISTICAL MECHANICS OF THE STEADY STATE. James A. McLennan, Jr., Department of Physics, Lehigh University, Bethlehem, Pennsylvania (Received April 28, 1959).

The description of steady-state phenomena, near equilibrium, in terms of Gibbs ensembles is discussed. The Liouville equation is modified to include external nonconservative forces which prevent the system from reaching equilibrium. The steady-state ensembles are then obtained as (approximately) time-independent solutions to the Liouville equation; such ensembles depend linearly on the thermodynamic parameters which characterize the deviation from equilibrium. With the aid of the steady-state ensembles the linear relations between the thermodynamic fluxes and forces are obtained.

NUCLEAR SPIN RELAXATION IN LIQUID He^3. R. H. Romer,* Physics Department, Duke University, Durham, North Carolina (Received April 20, 1959).

The nuclear spin thermal relaxation time, T_1, of He^3 nuclei has been measured between 1.2° and 3.0° in pure liquid He^3 under its saturated vapor pressure. The measured times are those characteristic of the bulk liquid. T_1 is 300 seconds at 1.2° and increases to 550 seconds at 3.0°. These results are in fair agreement with the theory of Bloembergen, Purcell, and Pound. For a 12% solution of He^3 in He^4 under its saturated vapor pressure, T_1 is about 2000 seconds and does not show any sharp change at the lambda point. In pure He^3 gas at 4.2°K and 1000 mm pressure, T_1 is at least 1000 seconds. In dilute solutions at 1.25°K, T_1 is at least 90 minutes for a 3.5% solution and at least 120 minutes for a 1.7% solution.

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ABUNDANCES OF THE RARE-EARTH NUCLEI PRODUCED BY RAPID NEUTRON CAPTURE IN SUPERNOVAE. Robert A. Becker* and William A. Fowler, Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California (Received April 16, 1959).

Calculations have been carried out, following the method of Burbidge, Burbidge, Fowler, and Hoyle for the abundances of nuclei in the rare-earth region which are produced in the rapid neutron-capture process thought to occur in supernovae. The recently available rare-earth mass differences of Johnson and Bhanot were employed. The calculated abundances agree, in general, with those given by Suess and Urey. The results of the computations support the work of Burbidge et al. which showed the effect of spheroidal deformation above the closed shell at $N=126$ in enhancing the production of Th^{232}, U^{235}, U^{238}, Cf^{254}, etc., in supernovae. The effect of different combinations of temperature and neutron density in enhancing certain relative abundances is discussed briefly.

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THEORY OF SUPEREXCHANGE. F. Keffer and T. Oguchi,* University of Pittsburgh, Pittsburgh, Pennsylvania (Received March 16, 1959).

The Dirac-Van Vleck-Serber spin operator expansion, first applied by Anderson to the Kramers superexchange problem, is extended, simplified, and systematized in order to handle

all overlap contributions arising from a number of interacting configurations. The linear cation-anion-cation (e.g., Mn^{++}-O^{--}-Mn^{++}) four-electron problem is worked out in detail, taking account of all contributions from configurations (*A*) ionic, (*B*) electron transferred to right, (*C*) electron transferred to left. Group symmetry requirements are invoked; and these, together with a simple approximation equivalent to perturbation theory, are shown to reduce the complicated matrix formulation to a single linear equation. The solution contains terms previously obtained by Anderson, by Anderson and Hasegawa, and by Yamashita, and a number of important extra terms. All superexchange terms are fourth order or higher in the overlap S. A rough numerical evaluation with modified Slater wave functions appropriate to MnO-type crystals yields an effective superexchange integral of the required size. Brief consideration is given to configurations in which two electrons are transferred, in particular (*D*) simultaneous transfer of electrons to right and to left (Slater mechanism). Unless the energy required to form this configuration is surprisingly small, its contribution is probably not so important, although the problem needs to be investigated in detail. Some consideration is also given to the linear cation-anion-anion-cation (e.g. Mn^{++}-Br^{-}-Br^{-}-Mn^{++}) problem; the formal solution for the ionic configuration is worked out; and it is shown that superexchange terms first appear in the order S^4T^2, where S is the anion-cation overlap and T is the anion-anion overlap.

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DENDRITIC GROWTH OF GERMANIUM CRYSTALS. A. I. Bennett and R. L. Longini, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received January 26, 1959; revised manuscript received August 13, 1959).

Controlled dendritic growth of germanium from the melt yields long thin strips whose principal surfaces are optically flat {111} crystallographic planes except for the occasional presence of small steps. The crystals grow rapidly in the ⟨211⟩ direction, have twin planes parallel to the flat surfaces, and are extremely strong mechanically. The distribution coefficients of impurities are close to unity.

A mechanism for dendritic growth is proposed, in which the presence of at least one properly oriented twin plane is fundamental and necessary. This mechanism explains most of the observed growth features in germanium dendrites, and is expected to apply generally to materials with the zincblende structure.

The presence of the twin plane makes growth in opposite directions in the twin plane dissimilar, not only in the zincblende lattice but generally. The effect of this asymmetrization on the growth of α-SiC is considered. A crystal growth mechanism based on this asymmetrization is proposed, which should be of general validity.

ENERGY LEVELS FOR RARE EARTH IONS SUBJECT BOTH TO EXCHANGE AND CRYSTALLINE FIELDS. Robert L. White and John P. Andelin, Jr., Hughes Aircraft Company, Culver City, California (Received April 1, 1959).

The term splitting has been calculated for the ground J state of the rare earth ions samarium through ytterbium for the case in which the ions are simultaneously subjected to exchange and crystalline fields of arbitrary relative magnitude. The crystalline field is taken to be cubic and as representing the potential of eight negative ions situated at the corners of a cube. The calculation is carried out for the magnetic axis in the two major (local) crystalline directions, [100] and [111]. The relevance of these calculations to the rare earth iron garnets is discussed.

LOW-TEMPERATURE ULTRASONIC ATTENUATION IN TIN AND ALUMINUM. Daniel H. Filson, Department of Physics, University of California, Los Angeles, California (Received April 14, 1959).

According to theory based on an ideal metal, the ultrasonic attenuation should be proportional to the square of the frequency and to the electrical conductivity of the metal. Experiments were performed to compare theoretical and experimental values of attenuation in a frequency range from 100 kc/sec to 1 Mc/sec.

A long wire sample coupled to a barium titanate transducer was suspended in a chamber, which in turn was placed in a liquid helium bath. A short train of sine waves was sent through the wire and the amplitude of successive reflections

was measured as a function of temperature and frequency. The electrical conductivity of the sample was also measured.

Two samples were investigated. High-purity tin yielded experimental results in excellent agreement with theory. The attenuation in high-purity aluminum was proportional to the electrical conductivity but averaged 45% higher than theoretically predicted.

NEW QUANTUM-MECHANICAL REPRESENTATION. E. C. McIrvine,* Physics Department, Cornell University, Ithaca, New York, and A. W. Overhauser,† Ramo-Wooldridge Corporation, Los Angeles, California, and Cornell University, Ithaca, New York (Received November 17, 1958).

The superlattice representation is described. This representation provides occupation probabilities which form a well-defined non-negative definite analog of the phase space density. The superlattice basis functions are orthonormal wave packets characterized in general by three parameters: a band index, a wave vector, and a superlattice position vector. The crystal momentum and position are automatically coarse-grained so as to satisfy the Heisenberg uncertainty principle. The Bloch and Wannier representations are special cases of the superlattice representation for particular choices of the superlattice parameter. Joint functions of position and momentum, such as the local current density, can be represented as the expectation value of Hermitian operators through the use of this representation.

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QUANTUM MECHANICAL APPROACH TO THERMAL TRANSPORT PHENOMENA IN METALS. E. C. McIrvine,* Physics, Department, Cornell University, Ithaca, New York (Received November 17, 1958; revised manuscript received June 12, 1959).

A quantum-mechanical transport equation corresponding to the classical Boltzmann equation is developed. Classical transport theory in the presence of a temperature gradient involves the phase-space density, which has no meaning in quantum mechanics due to the complementarity of position and momentum. The superlattice representation allows the development of a quantum transport equation from the density-matrix Schrödinger equation, through the introduction of irreversibility by standard methods. The quantum transport equation governing the electron occupation probability in the superlattice representation is derived for impurity scattering and for phonon scattering. This equation has the same field terms as the classical Boltzmann equation, but involves the discrete coarse-grained wave number and position of the superlattice representation. The scattering terms involve transitions between different superlattice states, and thus include scattering processes which move electrons from one superlattice cell to another. The solution of the classical equation is affected very little by these quantum effects, for all temperature gradients which might reasonably be encountered.

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DIPOLAR LINE BROADENING AND ENHANCED PSEUDO-DIPOLAR MOMENTS. Robert L. White, Hughes Research Laboratories, Culver City, California (Received May 5, 1959).

The assumption of a strongly enhanced pseudo-dipolar moment in certain ferrimagnetic and antiferromagnetic materials is shown to be incompatible with the paramagnetic resonance line widths in these materials. Why the concept of pseudo-dipolar moments is not applicable to line-width calculations is discussed.

ANOMALOUS SKIN EFFECT IN BISMUTH. George E. Smith,* Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received April 16, 1959).

High-frequency (23.5 kMc/sec) surface resistance measurements have been made on plane surfaces of single-crystal bismuth at 2°K as a function of orientation. It has been ascertained that extreme anomalous skin effect conditions prevail, allowing details of the Fermi surface to be deduced from Pippard's theory. In Shoenberg's model of the electron band, components of the inverse effective-mass tensor divided by the Fermi energy are found to be $\alpha_1/E_\ell = 9.10$, $\alpha_2/E_\ell = 0.088$, $\alpha_3/E_\ell = 4.7$, and $\alpha_4/E_\ell = 0.38$ (in units of

10^3/ev). These results are in essential agreement with values obtained from de Haas–van Alphen experiments and cyclotron resonance. The number of ellipses is definitely established to be six and the number of electrons found to be $N = 5.5 \times 10^{17}/\text{cm}^3$. The parameters for the two hole ellipsoids are found to be $\beta_1/E_h = \beta_2/E_h = 1.5$ and $\beta_3/E_h = 0.12$. Assuming Shoenberg's value $E_e = 17.7$ Mev, we calculate $E_h = 1.12$ Mev from specific heat data. It is also found that the reflection of carriers from the surface of the sample is predominantly specular in contrast to diffuse reflection found in other metals.

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STRUCTURE IN THE ENERGY DISTRIBUTION OF PHOTOELECTRONS FROM K_3Sb AND Cs_3Sb. E. A. Taft and H. R. Philipp, General Electric Research Laboratory, Schenectady, New York (Received April 27, 1959).

The energy distributions of photoelectrons from K_3Sb and Cs_3Sb show structure that is similar in form to structure in the spectral dependence of the optical absorption. One may rationalize this empirical result by saying that both the photoelectric and optical effects arise from structure in the state-density of the valence band. Assuming that the optical absorption involves transitions to the conduction band, a lower limit for the electron affinity of the crystal is 0.6 ev for K_3Sb and 0.4 ev for Cs_3Sb.

DIFFUSION MEASUREMENTS IN THE SYSTEM Cu-Au BY ELASTIC SCATTERING. Robert F. Sippel, Field Research Laboratory, Magnolia Petroleum Company, Dallas, Texas (Received April 24, 1959).

A broad-range magnetic spectrograph has been used to study the diffusion of gold into copper in the temperature range 360-500°C by elastic scattering of protons and deuterons. This is a new technique in solid diffusion measurements and should have fairly wide applicability in intermetallic diffusion. This investigation extends the data a factor of 10^6 below the sectioning range. These results, together with data of sectioning observers, show the Arrhenius law to be approximately valid over a range in D of 5×10^8. A precise fit to all the data requires a slight upward curvature. In the temperature range studied here, we obtain $Q = 45\,750 \pm 750$ cal/mole and $D_0 = 0.104 \pm 0.06$ cm^2/sec. This represents a decrease of about 4000 cal/mole from the most precise data in the sectioning range. This curvature is attributed to diffusion along internal surfaces, although the possibility of multiple-diffusion mechanisms still remains.

ACOUSTIC ATTENUATION IN ALUMINUM DUE TO ELECTRON-LATTICE INTERACTION. Edward Lax, Department of Physics, University of California, Los Angeles, California (Received April 22, 1959).

The attenuation of sound due to the interaction of electrons with the lattice has been measured in pure polycrystalline aluminum at low temperatures. Measurements from 26 to 130 kc/sec, obtained by recording the decays of the harmonics of a longitudinally resonating bar, were compared with the electrical conductivity of an aluminum wire of identical purity, over a temperature range of 3° to 70°K. The attenuation was proportional to the square of the frequency. A detailed agreement between the shape of the attenuation and conductivity curves was obtained, but the experimental attenuation was 50% greater than could be explained by the present theory.

THEORY OF FERROMAGNETIC RESONANCE IN RARE EARTH GARNETS. I. g-VALUES. C. Kittel, Department of Physics, University of California, Berkeley, California, and Hughes Research Laboratories, Culver City, California (Received April 27, 1959).

Ferromagnetic resonance phenomena in rare earth iron garnets (except gadolinium iron garnet) are dominated at room temperature and above by the exceedingly rapid relaxation of rare earth ions. The rare earth ion relaxation controls the g-values and the line widths, as well as the temperature dependence of these quantities. It is shown that in the appropriate limit the g-value of the microwave spin resonance line satisfies the relation $g = g_A(M_A + M_B)/M_A$, where g_A and M_A refer to the ferric lattice and $M_A + M_B$ is the net saturation magnetization of the crystal. This relation obtains essentially when the B lattice relaxation frequency is high in comparison with

the AB exchange frequency and relaxation frequency of the A lattice. The theory accounts quite well, with no disposable parameters, for the sequence and temperature variation of the g-values reported for Dy, Ho, Er, Yb, and Sm iron garnets. When at low temperatures the B relaxation frequency becomes sufficiently low, the g-value should approach the usual result for two coupled undamped lattices, in the absence of anisotropy effects on the B lattice. The theory predicts further that at ordinary temperatures the exchange frequency resonance will occur at the usual position and its width will be proportional to the damping constant of the B lattice.

FERROMAGNETIC SOLUTES IN SUPERCONDUCTORS. Bernd Matthias, Vera B. Compton, Harry Suhl, and Ernest Corenzwit, Bell Telephone Laboratories, Murray Hill, New Jersey (Received April 27, 1959).

Solid solutions of chromium, manganese, iron, or cobalt in titanium raise its superconducting transition temperature by almost an order of magnitude. At very small concentrations this is much more than could be expected from a variation of electron concentration.

SATURATION OF THE PARAMAGNETIC RESONANCE OF A V CENTER. T. G. Castner, Jr.,* Physics Department, University of Illinois, Urbana, Illinois (Received April 16, 1959).

The saturation of the paramagnetic resonance of the $(\text{halogen})_2^-$ complex (V_K center) in the alkali halides has been studied. The saturation of the absorption signal ($\chi''H_1$) versus H_1 is obtained over a 60-db power range for KCl, KBr, and LiF at 78°K. Portis' theory of inhomogeneous saturation has been generalized by omitting the assumption that the individual spin packet width is very much smaller than the envelope width. Methods are developed to determine independently, from a given experimental saturation curve, the spin packet width $1/T_2$, the spin-lattice relaxation time T_1, and the product T_1T_2. For KCl and LiF values of T_2, T_1, and T_1T_2 are determined for the different hyperfine lines of the V_K-center spectrum. For KBr, only the product T_1T_2 could be obtained. From the results it is concluded that the spin packet width is not limited by T_1. For KCl at 78°K, $T_1 \sim 7T_2$; for LiF, T_2 is two orders of magnitude or more less than T_1 and depends in a complicated manner on the external magnetic field and the angle between the V_K-center axis and the external field.

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DEPENDENCE OF THE HOLE IONIZATION ENERGY OF IMPERFECTIONS IN CADMIUM SULFIDE ON THE IMPURITY CONCENTRATION. Richard H. Bube and Arthur B. Dreeben, RCA Laboratories, Radio Corporation of America, Princeton, New Jersey (Received April 17, 1959).

The variation of the hole ionization energy of imperfections in cadmium sulfide as a function of the impurity concentration has been measured using photoconductivity in a series of CdS:Ga:Cu powders. The Cu concentration varies from 4×10^{17} to 2×10^{20} cm^{-3}, and each sample was prepared with a Cu-concentration to Ga-concentration ratio of 1.05. The hole ionization energy of the sensitizing centers, as determined from the thermal quenching of photoconductivity, decreases from about 1.0 ev for low Cu concentrations to about 0.3 ev for 2×10^{20} Cu cm^{-3}. The results are analogous to other recent findings of small hole ionization energies in CdS and CdSe crystals.

MAGNETIZATION OF COMPOUNDS OF RARE EARTHS WITH PLATINUM METALS. R. M. Bozorth, B. T. Matthias, H. Suhl, E. Corenzwit, and D. D. Davis, Bell Telephone Laboratories, Murray Hill, New Jersey (Received April 30, 1959).

Measurements have been made of the Curie points and magnetic moments of three series of compounds (Laves phases) $M\text{Ir}_2$, $M\text{Os}_2$, and $M\text{Ru}_2$, where M is a rare-earth element. The Curie point is highest (85°K) for the compounds containing Gd and falls away when the rare earths have larger or smaller atomic numbers. This dependence of Curie point on the rare-earth element is in accord with the idea that the principal exchange interaction is between the spins of the $4f$ shell of the rare-earth ions and the conduction electrons. The magnetic moments are closely related to, but differ somewhat from, the values of Jg for the trivalent ions of M.

BAND STRUCTURE OF NOBLE METAL ALLOYS: OPTICAL ABSORPTION IN α BRASSES AT 4.2°K. M. A. Biondi and J. A. Rayne, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received April 22, 1959).

Calorimetric optical absorption measurements at 4.2°K have been made on a representative series of α brasses over the wavelength range 0.23 to 4.0 microns using electropolished bulk specimens. Changes in the absorption spectrum below 6000 A are interpreted in the light of current theories concerning the band structure of noble metal alloys. The variation of the infrared absorptivity with residual resistivity shows that the impurity relaxation time for copper is anisotropic. This anisotropy increases on alloying and suggests that the Fermi surface becomes more distorted with increasing solute concentration.

AGGREGATION OF F CENTERS IN POTASSIUM IODIDE. D. H. Goode and P. A. Schroeder,* Physics Department, University of Canterbury, Christchurch, New Zealand (Received April 23, 1959).

As optical aggregation of F centers proceeds in KI the R, M, and N bands develop, but later lose their identity in a very broad band, which is absent in KCl and KBr. The shape and position of this band are temperature independent, and it is removed from the peak of the true colloid band. A new band on the short-wavelength side of the F band was observed in crystals containing high concentrations of F centers.

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NATURE OF THE SINGULARITIES IN THE SPECTRUM OF A ONE-DIMENSIONAL IONIC LATTICE. Joseph Gillis and George Weiss,* Department of Applied Mathematics, Weizmann Institute of Science, Rehovot, Israel (Received April 6, 1959).

It is known, by a theorem of van Hove, that regular lattices will have singularities in their frequency spectra, the type of singularity depending on the dimension. Although several studies of the one-dimensional lattice with Coulomb interactions have appeared, no one yet has succeeded in identifying the type of singularity that appears in addition to the inverse square root singularity, in the frequency spectrum. In this note we establish that for a one-dimensional crystal the singularity is of the form $g(\omega) \sim A\{(\omega-\omega_1)\ln[1/(\omega-\omega_1)]\}^{-1/2}$, where A is a constant and ω_1 is the position of the singularity.

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PROPERTIES OF ALKALI HALIDE CRYSTALS AT LOW TEMPERATURES. T. H. K. Barron, Department of Physical and Inorganic Chemistry, The University, Bristol, England, and J. A. Morrison, Division of Pure Chemistry, National Research Council, Ottawa, Canada (Received March 13, 1959).

Recent experimental results on low-temperature heat capacities and elastic constants of alkali halide crystals are examined for evidence of anharmonic contributions. It is concluded that if such effects are present, their contribution to the characteristic temperature at 0°K is less than that of uncertainties in the experimental results.

POSITRON ANNIHILATION IN HELIUM. Thomas Bruce Daniel* and Robert Stump, Physics Department, University of Kansas, Lawrence, Kansas (Received April 27, 1959).

Positron mean lives were measured in helium gas over a wide temperature range at densities from 4.6 to 534 times the STP density and in liquid helium at 4.2° and 5.1°K. The orthopositronium mean life in the liquid at 5.1°K was found to be $(9.4 \pm 0.6) \times 10^{-8}$ sec which differs little from the lifetime in liquid at 4.2°K. Except at very low temperatures, the pick-off rate in gas agreed moderately well with theory. The low pick-off rate associated with orthopositronium in liquid was observed also in the gas at temperatures below 9°K for densities greater than 250 times the STP density.

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ALPHA-DECAY BARRIER PENETRABILITIES WITH AN EXPONENTIAL NUCLEAR POTENTIAL. II. ODD-MASS NUCLEI. John O. Rasmussen, Lawrence Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received April 17, 1959).

Calculation of barrier penetrabilities, reduced widths, and hindrance factors for odd-mass alpha-particle emitters is made by using the diffuse exponential nuclear potential derived from optical-model analysis of alpha-particle elastic scattering data. The calculations are made on the same basis as for even-even alpha emitters, reported in a previous publication.

NUCLEAR BAND STRUCTURE IN Sc^{41}. R. H. Davis, Department of Physics, Florida State University, Tallahassee, Florida (Received April 29, 1959).

Band structure predictions assuming the formation of proton single-particle states above the Ca^{40} core in the ground state or one of its excited states are compared with the available data on the elastic and inelastic scattering of protons from Ca^{40}. The band expected above the 3.35-Mev state in Ca^{40} is confirmed by experimental results, and some evidence is found for bands above the higher core states.

ELASTIC SCATTERING OF 20.35-Mev PROTONS BY Zn^{64}, Zn^{66}, AND Zn^{68}. R. W. Boom* and J. Reginald Richardson, Department of Physics, University of California, Los Angeles, California (Received April 27, 1959).

The absolute differential cross section for the elastic scattering of 20.35 ± 0.25 Mev protons has been measured for enriched Zn^{64}, Zn^{66}, and Zn^{68} foils. In the angular range of 30°-160°, about 50 measurements were made for each foil (spaced from 1° to 5°) to an estimated accuracy of about 5% standard deviation. Scattered protons were detected by nuclear emulsions wrapped around a 4-in. diameter scattering chamber, all angles being exposed simultaneously. Detector energy resolution is 2.5%, angular resolution is 1° standard deviation, and relative angular shifts are determined to 0.1°. Correction has been made for the finite sizes of beam and detector and for multiple scattering in the target and in the detector stopper. The ±0.25-Mev energy spread includes maximum and minimum energies due to beam drift, beam spread, and target foil thickness. Three minima are found for each isotope: Zn^{64} at 63°, 104°, 142°; Zn^{66} at 62°, 102.5°, 142°; and Zn^{68} at 61°, 101°, 142°. The absolute cross sections are approximately the same except at the third minima, where for Zn^{64}, Zn^{66}, and Zn^{68} they are, respectively, 1.46, 1.07, and 0.61 mb/sterad.

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DECAY SCHEMES OF THE ISOMERS OF Tc^{95} AND Tc^{97}. John P. Unik and John O. Rasmussen, Lawrence Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received March 30, 1959).

An investigation of the decay schemes of Tc^{95} and Tc^{97} has been made by using high-resolution conversion-electron spectrographs, gamma-ray scintillation detectors, and coincidence techniques.

In addition to the 38.9-kev isomeric transition in Tc^{95}, eight transitions of the following energies have been assigned to Mo^{95}: 204.2, 583.9, 763, 767.9, 784, 788.0, 822.5, and 837.3 kev. A decay scheme is proposed.

The isomer of Tc^{97} has been shown to decay by a single $M4$ transition of 96.5 ± 0.1 kev. The experimental $K:L:M$ relative conversion-electron intensities for this transition are 1:0.48:0.13.

ANGULAR DISTRIBUTIONS OF PROTONS FROM THE (d, p) REACTION WITH DEUTERON ENERGIES BELOW THE COULOMB BARRIER. J. P. Schiffer and L. L. Lee, Jr., Argonne National Laboratory, Lemont, Illinois (Received April 23, 1959).

Cross sections and angular distributions for the (d, p) reaction to a known $l=1$ single-particle state of the captured neutron have been measured for seven target nuclides between Ti and Ni. Targets with thicknesses of several hundred kev were used with deuteron bombarding energies of 3.8 and 4.5 Mev. The angular distributions were found to be similar, with approximately 2:1 forward peaking and a broad maximum at about 60°. In addition, a sharp but relatively weak peak was observed at about 25° for the

lighter of the target nuclides. This is the angle at which the theory of Butler would predict a maximum for neutron capture with $l_n = 1$. Analysis of the proton spectra and angular distributions indicates that compound nucleus formation contributes less than 25% to the reaction yield at these deuteron energies.

BINDING-EFFECT CORRECTIONS IN THE ENERGY-LOSS DISTRIBUTION FUNCTIONS FOR CHARGED PARTICLES PASSING THROUGH THIN ABSORBERS. Walter Rosenzweig, Department of Radiology, Columbia University, New York, New York (Received April 24, 1959).

In the theoretical treatment of the statistical fluctuation in energy loss of charged particles passing through thin absorbers, terms which are of the order of the electron binding energy are ordinarily neglected. The corrections required, when this is no longer justified, are discussed in particular reference to the development by Symon, which has as its limiting cases the Gaussian distribution derived by Bohr and Landau's distribution. Formulas are presented which give the correction for each of the three weighted parameters appearing in Symon's derivation. The corrected parameters can then be used in conjunction with Symon's curves to obtain the energy-loss distribution appropriate for a given set of conditions. An illustration is presented for the cases of 5-Mev protons and 5-Mev alpha particles.

ELASTIC SCATTERING OF N^{14} BY Be^9. M. L. Halbert and A. Zucker, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received April 20, 1959).

Nitrogen-beryllium elastic scattering was measured over an angular range from 32 to 144 deg in the center-of-mass system with an angular resolution of about one degree. The mean energy of the incident nitrogen ions was 27.3 Mev. To distinguish elastic scattering from other events both the scattered and the recoil particles were detected in coincidence by thin CsI(Tl) scintillation counters. The elastic scattering differential cross section is 550 mb/sterad at 32 deg c.m. It decreases monotonically and more rapidly than $\csc^4(\theta/2)$ to a shallow minimum of about 5 mb/sterad at 106 deg c.m., rises slightly, and then falls to about 2.5 mb/sterad at 144 deg c.m., the largest angle measured. The data are compared to the predictions of a sharp cutoff model for elastic scattering, but no agreement is found between this theory and the experimental results.

CROSS SECTION FOR THE $Li^6(n,\alpha)H^3$ REACTION FOR $1.2 \leq E_n \leq 8.0$ Mev. R. B. Murray and H. W. Schmitt, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received April 17, 1959).

The cross section for the reaction $Li^6(n,\alpha)H^3$ has been measured as a function of neutron energy in the range $1.2 \leq E_n \leq 8.0$ Mev. An essentially back-to-back method was used, with a thin cylindrical $Li^6I(Eu)$ scintillation crystal placed concentric with and adjacent to a thin-walled ionization chamber containing a deposit of fissile material (U^{238} or Np^{237}). The magnitude of $\sigma_{n,\alpha}$ as measured in this experiment depends on the absolute value of $\sigma_{\text{fiss}}(U^{238})$, while the shape of the cross-section vs energy curve depends on the known energy dependence of $\sigma_{\text{fiss}}(U^{238}$ or $Np^{237})$. Statistical and other point-to-point uncertainties in the data range from ±5 to ±9%, while the uncertainty in absolute value of the cross section is ±7%. The cross section obtained from these measurements decreases monotonically from a value of 0.28 barn at $E_n = 1.2$ Mev to 0.051 barn at $E_n = 8.0$ Mev.

ENERGY LEVELS IN A BOUNDED ISOTROPIC HARMONIC OSCILLATOR POTENTIAL AND NUCLEAR SHELL STRUCTURE. S. Sengupta and S. Ghosh, Physics Department, Hooghly Mohsin College, Hooghly (W.B.), India (Received March 14, 1958).

Energy levels for a three-dimensional bounded harmonic oscillator are obtained for intermediate distances of the boundary. Energy levels are found to depend on the dimensionless parameter $\rho_0 = (\omega M/\hbar)R^2$. ρ_0 is the ratio of the strength of the oscillator levels ($\hbar\omega$) to that of the square well levels ($\hbar^2/MR^2$).

Adding a spin-orbit energy 30 times the Thomas value, nuclear energy levels are worked out for the two mass regions at $A = 50$ and 90. For $\rho_0 = 6$, good agreement with the experimental level sequence is obtained.

POLARIZATION OF γ RAYS IN Pb^{207}. P. H. Stelson, W. G. Smith, and F. K. McGowan, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received April 20, 1959).

Polarization-direction measurements have been made on the γ-ray transitions in Pb^{207} resulting from the decay of Bi^{207}. These measurements were made to obtain additional information on both the spin of the level at 2.34 Mev and the multipole character of the decay γ ray. The observed polarization eliminates the possible spin assignment 9/2. It is concluded that the spin assignment is 7/2 and that the 1.77-Mev γ ray is predominantly $M1$.

TOTAL REACTION AND ELASTIC SCATTERING CROSS SECTIONS FOR 22.8-Mev PROTONS ON URANIUM ISOTOPES. Clyde B. Fulmer, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received April 16, 1959).

The total reaction cross sections for 22.8-Mev protons on uranium isotopes was determined by measuring cross sections for $(p,\text{fission})$, (p,p'), (p,d), (p,t), and (p,α). These data are combined with previously measured (p,xn) cross sections to give total reaction cross sections of 1.43 ± 0.10, 1.44 ± 0.10, and 1.39 ± 0.10 barns for 22.8-Mev protons on U^{233}, U^{235}, and U^{238}, respectively. Angular distributions of (p,p'), (p,d), (p,t), (p,α), and proton elastic scattering cross sections are shown for U^{235} and U^{238}

STRANGE-PARTICLE PRODUCTION BY BEVATRON NEUTRONS IN PROPANE. Charles O. Dechand,* Sloane Physics Laboratory, Yale University, New Haven, Connecticut (Received April 24, 1959).

A liquid propane bubble chamber was exposed to a beam of neutrons with energies up to 6 Bev from the Bevatron. 10 000 pictures of interactions in the hydrocarbon were scanned to detect neutral heavy unstable particles. 349 neutral V events were found, most of which came from the stainless steel walls of the chamber. 86% of these events could be identified as one or the other or either of the neutral strange particles: Λ^0 or Θ_1^0. The Λ^0/Θ_1^0 ratio is about 0.6.

8200 stars of two or more prongs formed by neutrons interacting in the liquid propane were observed in the chamber and 17 of these produced V^0's. An additional 5 V^0's were formed in single-prong events produced by neutrons, and 8 others were produced in events in the propane caused by charged particles.

The energy spectrum of the incident neutrons was estimated from study of π-meson production interactions in the hydrogen. The distribution shows that the neutrons had energies up to 6 Bev with a mean value of about 4 Bev. For the energy range 1 to 6 Bev, the production of strange particles occurs in about 1% of all inelastic interactions of neutrons with hydrogen and carbon.

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K^+-DEUTERON SCATTERING IN THE IMPULSE APPROXIMATION. Erasmo M. Ferreira,* Imperial College of Science and Technology, London, England (Received September 17, 1959; revised manuscript received March 25, 1959).

It is suggested that a phenomenological analysis using the impulse approximation of the processes occurring in the scattering of K^+ mesons by deuterons may be used to get the phase shifts for the $T=0$ isotopic spin state. Typical curves are given for the elastic, elastic plus inelastic, and charge exchange scattering differential cross sections of 100-Mev K^+ mesons by deuterons on the assumption that only S waves contribute and for various ratios of the $T=1$ and $T=0$ isotopic spin state phase shifts.

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MEASUREMENT OF PRIMARY DIRECTIONS IN EXTENSIVE AIR SHOWERS. C. B. A. McCusker, Dublin Institute for Advanced Studies, Dublin, Ireland (Received March 24, 1958).

The abilities of various extensive shower arrays to detect possible departures from isotropy of the directions of incidence of very high-energy cosmic-ray primaries are analyzed. The effects of angular resolution, selectivity of total energy, and sensitivity to the nature of the primary are discussed. A simple method of analyzing variation in intensity with declination is given and applied to some recent experimental results. It is shown that in two well-known cases these are

not consistent with the hypothesis of isotropy of incoming directions.

DIRECTIONAL PROPERTIES OF AN EXTENSIVE AIR SHOWER ARRAY. C. B. A. McCusker, D. E. Page, and R. A. Reid, Dublin Institute for Advanced Studies, Dublin, Ireland (Received March 24, 1958).

The directional properties of an extensive air shower array consisting of three Geiger-Müller counter telescopes have been examined. It is shown that the directional properties of this array show no great improvement over more conventional arrangements. Possible ways of improving the performance of the device are suggested.

HEAVY PRIMARY COSMIC RAYS AT GEOMAGNETIC LATITUDE OF 41°N. O. B. Young and H. Y. Chen, Southern Illinois University, Carbondale, Illinois (Received April 21, 1959).

This report includes the results from 9 balloon flights at geomagnetic latitude 41°N of altitude range from 70 000 to 100 000 feet. Only primaries of $Z \geq 10$ are considered. 2410 tracks are involved in Ilford G-5 and G-0 emulsion exposures. Given are the charge spectra, flux, mean free paths, and angular distributions.

NUCLEAR INTERACTIONS AT ENERGIES OVER 1000 Bev. J. G. McEwen,* Division of Pure Physics, National Research Council, Ottawa, Canada (Received April 24, 1959).

Measurements on two jets, a $0+36p$ jet with primary energy about 2500 Bev and a $4+29\alpha$ jet with primary energy about 8000 Bev/nucleon, are presented. It is shown that the main features of the angular distribution of particles from these two primary jets and a third energetic secondary jet can be explained satisfactorily by the model in which mesons created in a nucleon-nucleon collision are considered to be radiated isotropically from two centers. Further implications of this model are discussed; in particular it is shown that definite restrictions are imposed on the values of energy and angle of emission of secondary particles; the average transverse momentum is predicted to be relatively constant but to have a lower value not only in the forward and backward directions but also around 90° in the center-of-mass system of the two colliding nucleons. While it is shown that the available experimental data are in accord with these predictions, more events with energies in the region of 1000 Bev must be studied before a definite conclusion can be reached.

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PRIMARY COSMIC-RAY PROTON AND ALPHA-PARTICLE INTENSITIES AND THEIR VARIATION WITH TIME. Peter Meyer, The Enrico Fermi Institute for Nuclear Studies and Department of Physics, The University of Chicago, Chicago, Illinois (Received April 22, 1959).

A series of high-altitude balloon flights was carried out in 1957 and 1958 to study the flux of primary cosmic-ray protons and α particles during variations in the total cosmic ray intensity. The following results are obtained for α particles with energies exceeding 530 Mev/nucleon under 13.5 g/cm² of air: (a) During a large Forbush-type decrease the α-particle and proton intensities were closely correlated. This demonstrates that a modulation mechanism is operating on both components. (b) At certain times, variations in the α-particle intensity were observed within a few hours which were not accompanied by corresponding changes in the proton flux. This is tentatively ascribed to an anisotropy in the α-particle flux that reaches the earth. (c) While there existed an intensity decrease in the proton flux between 1957 and 1958 which is also observed in the neutron monitor station data, no such variation occurred in the α-particle flux. A division of the α particles into two energy groups (450 Mev/nucleon $\leq E_1 \leq 960$ Mev/nucleon and $E_2 \geq 960$ Mev/nucleon) shows: (a) that the Forbush decrease is of the same magnitude in both energy groups, (b) that the hourly flux increase observed in some flights is about the same in both energy groups, and (c) that from 1957 to 1958 the flux in the low-energy group increased, while it decreased in the high-energy interval, contrary to the well-known behavior of the proton flux. These independent α-particle flux variations cannot be explained by any of the modulation mechanisms so far proposed. We suggest that occasional solar production of α particles may be responsible for our results.

The absolute flux of α particles with energies exceeding 560 Mev/nucleon at the top of the atmosphere was measured on five different days.

ANALYTIC PROPERTIES OF TRANSITION AMPLITUDES IN PERTURBATION THEORY. Stanley Mandelstam, Department of Physics, University of California, Berkeley, California (Received April 17, 1959).

The analytic properties of two-particle transition amplitudes as functions of both energy and momentum transfer are examined in perturbation theory. The modified Nambu representation previously proposed by the author for expressing these properties is discussed in a little more detail. It is shown that, as long as the masses do not satisfy certain inequalities connected with the existence of anomalous thresholds, the fourth-order terms, calculated in the usual manner, satisfy the representation. The spectral functions are calculated explicitly for spinless particles. The proof can be extended to the sixth order, but is not worked out here. The modifications necessary when there exist anomalous thresholds are mentioned.

CONSTRUCTION OF THE PERTURBATION SERIES FOR TRANSITION AMPLITUDES FROM THEIR ANALYTICITY AND UNITARITY PROPERTIES. Stanley Mandelstam, Department of Physics, University of California, Berkeley, California (Received April 17, 1959).

The analyticity properties of transition amplitudes are used in conjunction with the unitarity requirements to generate successive terms in the perturbation series, without referring to a specific Lagrangian. In the sixth and higher orders, production is neglected in the unitarity condition; subject to this approximation, it is found that the series can be so constructed. No analyticity properties which have not been rigorously proved need be employed, and the terms are found to satisfy the double dispersion representation. By examining the connection between this method and the conventional calculation of the perturbation series the types of spectral function corresponding to different Feynman diagrams can be found. Formulas are given for the regions in which the spectral functions are nonzero.

TEST OF GLOBAL SYMMETRY IN K-p REACTIONS. Marc H. Ross and Gordon L. Shaw, Indiana University, Bloomington, Indiana (Received April 17, 1959; revised manuscript received July 21, 1959).

A method of testing the hypothesis that there is global symmetry of the pion-baryon interaction is proposed: Upon analyzing low energy K^--p scattering data, one finds a variety of scattering-length solutions which are compatible with the elastic scattering, charge-exchange scattering, and total charged hyperon production. Our suggestion involves the use of the experimental Σ^+/Σ^- ratios to (a) test the global symmetry hypothesis (or any other quantitative description of the pion-hyperon interaction), (b) reduce the ambiguity in the $\overline{K}$-N scattering-length solutions, (c) hence, predict the Σ^0 cross section. We need to know the π - Y phase shifts, in addition to the $\overline{K}$-N scattering lengths, in order to predict the Σ^+, Σ^- cross sections. If there is global symmetry of pion-baryon interactions, then we know the π - Y phases in the absence of a $\overline{K}$-N reaction channel. We demonstrate how the actual π - Y phase shifts can be obtained, in a non-perturbative manner, from the idealized π - Y phases (i.e., in the absence of $\overline{K}$-N reactions) and the $\overline{K}$-N scattering lengths. Certain fits to the present rough data with scattering lengths of negative real part are shown to be incompatible with global symmetry. Earlier proposals making use of the dependence of the hyperon production ratios on the π - Y phase shifts are also examined in terms of this result. The proposed analysis involves the assumption that the $\overline{K}$ is an isotopic doublet.

PROPOSAL FOR DETERMING THE ELECTROMAGNETIC FORM FACTOR OF THE PION. William R. Frazer, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received April 23, 1959).

The possibility of measuring the electromagnetic form factor of the pion by extrapolation of the cross section for $e^- + p \to n + \pi^+ + e^-$ has been investigated. The method is based on the existence of a pole in the pion electroproduction scattering amplitude as a function of the invariant momentum transfer of the nucleon. The residue of this pole is the pion form factor multiplied by a known coefficient. Since the pole

lies slightly outside the physical region of the invariant momentum transfer, an extrapolation of the experimental data is required. An approximate calculation of the pion electroproduction cross section has been made in order to estimate the experimental accuracy necessary for a significant extrapolation. Accuracy is required which is an order of magnitude better than that achieved at present in similar experiments.